

# Inverse and Composite Functions

## Mark Scheme

Level	GCSE
Subject	Maths
Exam Board	Edexcel GCSE
Topic	Inverse and Composite Functions
Grade Level	Grade 7
Booklet	Mark Scheme

**Time Allowed:** 53 minutes

**Score:** /44

**Percentage:** /100

**Grade Boundaries:**

1. Given that  $f(x) = x - 4$  find:

a)  $f(5)$

$$f(5) = 5 - 4$$

$$\dots\dots\dots 1 \dots\dots\dots (1)$$

b)  $f(3)$

$$f(3) = 3 - 4$$

$$\dots\dots\dots -1 \dots\dots\dots (1)$$

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2. Given that  $g(x) = 2x^2 - 10$  find:

a)  $g(2)$

$$\begin{aligned} g(2) &= 2(2)^2 - 10 \\ &= 8 - 10 \end{aligned}$$

$$\dots\dots\dots -2 \dots\dots\dots (1)$$

b)  $g(-2)$

$$\begin{aligned} g(-2) &= 2(-2)^2 - 10 \\ &= 8 - 10 \end{aligned}$$

$$\dots\dots\dots -2 \dots\dots\dots (1)$$

c) Solve:  $g(x) = 8$

$$2x^2 - 10 = 8$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$\dots\dots\dots x = \pm 3 \dots\dots\dots (3)$$

3. Given that  $f(x) = 3x - 5$  find:

a)  $f(3) = 3(3) - 5$   
 $= 9 - 5$

.....<sup>4</sup>..... (1)

b)  $f(-2) = 3(-2) - 5$   
 $= -6 - 5$

.....<sup>-11</sup>..... (1)

c) Solve:  $f(x) = 1$

$$\begin{aligned} 3x - 5 &= 1 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

..... (2)

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4. Given that  $f(x) = x^2 - 3$  find:

a)  $f(10) = (10)^2 - 3$   
 $= 100 - 3$

.....<sup>97</sup>..... (1)

b)  $f(-1) = (-1)^2 - 3$   
 $= 1 - 3$

.....<sup>-2</sup>..... (1)

c) Find:  $f^{-1}(x)$

$$\begin{aligned} y &= x^2 - 3 \\ y + 3 &= x^2 \\ \sqrt{y + 3} &= x \end{aligned}$$

$$f^{-1}(x) = \sqrt{x + 3}$$

$f^{-1}(x) = \sqrt{x + 3}$  ..... (2)

5. Given that  $f(x) = 2x - 4$  and  $g(x) = 3x + 5$

a) Find:  $gf(3)$

$$\begin{aligned} f(3) &= 2(3) - 4 \\ &= 6 - 4 \\ &= 2 \end{aligned}$$

$$\begin{aligned} g(2) &= 3(2) + 5 \\ &= 6 + 5 \end{aligned}$$

..... (2)

b) Work out an expression for:  $f^{-1}(x)$

$$y = 2x - 4$$

$$y + 4 = 2x$$

$$\frac{1}{2}(y + 4) = x$$

$$f^{-1}(x) = \frac{1}{2}(x + 4)$$

$f^{-1}(x) = \frac{1}{2}(x + 4)$  (2)

c) Solve:  $f(x) = g(x)$

$$2x - 4 = 3x + 5$$

$$-4 = x + 5$$

$$x = -9$$

$x = -9$  (2)

6. Given that  $f(x) = 3x + 1$  and  $g(x) = x^2$

a) Write down an expression for:  $fg(x)$

$$3x^2 + 1 \dots (2)$$

b) Work out an expression for:  $gf(x)$

$$(3x + 1)^2 \dots (2)$$

c) Solve:  $fg(x) = gf(x)$

$$3x^2 + 1 = (3x + 1)^2$$

$$3x^2 + 1 = 9x^2 + 3x + 3x + 1$$

$$0 = 6x^2 + 6x$$

$$0 = x^2 + x$$

$$0 = x(x + 1)$$

$$x = 0 \quad x = -1$$

$$x = 0 \quad x = -1 \dots (3)$$

7. Given that  $f(x) = x^2 - 17$  and  $g(x) = x + 3$

a) Work out an expression for:  $g^{-1}(x)$

$$y = x + 3$$

$$y - 3 = x$$

$$g^{-1}(x) = x - 3 \quad (2)$$

b) Work out an expression for:  $f^{-1}(x)$

$$y = x^2 - 17$$

$$y + 17 = x^2$$

$$\sqrt{y + 17} = x$$

$$f^{-1}(x) = \sqrt{x + 17} \quad (2)$$

c) Solve:  $f^{-1}(x) = g^{-1}(x)$

$$\sqrt{x + 17} = x - 3$$

$$x + 17 = (x - 3)^2$$

$$x + 17 = x^2 - 6x + 9$$

$$0 = x^2 - 7x - 8$$

$$0 = (x - 8)(x + 1)$$

$$x = 8 \quad x = -1$$

..... (4)

8. A function  $f$  is defined such that

$$f(x) = x^2 - 1$$

a) Find an expression for:  $f(x-2)$

$$\begin{aligned} f(x-2) &= (x-2)^2 - 1 \\ &= x^2 - 2x - 2x + 4 - 1 \\ &= x^2 - 4x + 3 \end{aligned}$$

$$\underline{\underline{x^2 - 4x + 3}} \quad (2)$$

b) Hence solve:  $f(x-2) = 0$

$$\begin{aligned} x^2 - 4x + 3 &= 0 \\ (x-3)(x-1) &= 0 \\ x=3 \quad x=1 \end{aligned}$$

$$\underline{\underline{x=3 \quad x=1}} \quad (2)$$

9. A function  $f$  is defined such that

$$f(x) = 4x - 1$$

a) Find:  $f^{-1}(x)$

$$y = 4x - 1$$

$$y + 1 = 4x$$

$$\frac{y+1}{4} = x$$

$$f^{-1}(y) = \frac{y+1}{4}$$

..... (2)

The function  $g$  is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant}$$

Given that  $fg(2) = 12$

b) Work out the value of  $k$

$$\begin{aligned} g(2) &= k(2)^2 \\ &= 4k \end{aligned}$$

$$\begin{aligned} f(4k) &= 4(4k) - 1 \\ &= 16k - 1 \end{aligned}$$

$$16k - 1 = 12$$

$$16k = 13$$

$$k = \frac{13}{16}$$

..... (2)