

Perpendicular Lines and the equation of a tangent

Mark Scheme

Level	GCSE
Subject	Maths
Exam Board	Edexcel GCSE
Topic	Perpendicular Lines and the equation of a tangent
Grade Level	Grade 8/9
Booklet	Mark Scheme

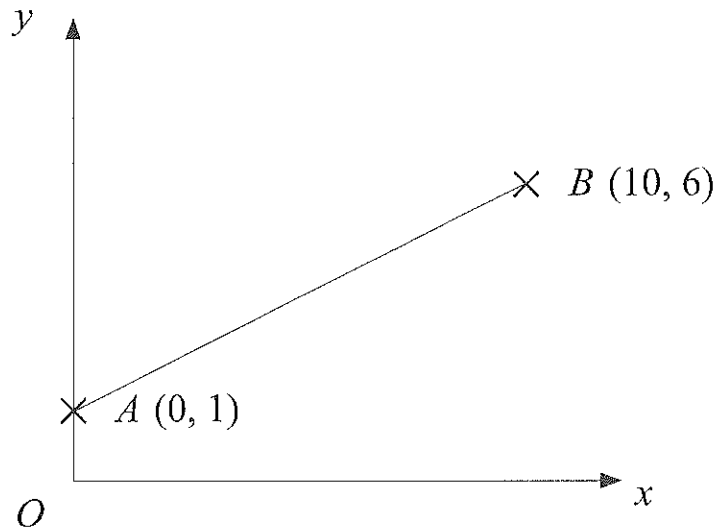
Time Allowed: 48 minutes

Score: /40

Percentage: /100

Grade Boundaries:

1.



A is the point (0, 1)

B is the point (10, 6)

The equation of the straight line through A and B is $y = \frac{1}{2}x + 1$

a) Write down the equation of another straight line parallel to $y = \frac{1}{2}x + 1$

$$y = \frac{1}{2}x \dots \dots \dots (1)$$

b) Write down the equation of another straight line that passes through the point (0, 1)

$$y = 5x + 1 \dots \dots \dots (1)$$

c) Find the equation of the line perpendicular to AB passing through B.

$$AB \text{ Gradient} = \frac{1}{2}$$

$$\text{Perp. gradient} = -2$$

$$\begin{pmatrix} 10 & 6 \\ x & y \end{pmatrix}$$

$$y = -2x + c$$

$$6 = -2(10) + c$$

$$6 = -20 + c$$

$$c = 26$$

$$y = -2x + 26 \dots \dots \dots (3)$$

2.

A straight line, L, passes through the point with coordinates (4, 7) and is perpendicular to the line with equation $y = 2x + 3$.

$$m=2$$

Find an equation of the straight line L.

$$\text{perp. gradient} = -\frac{1}{2}$$

$$\begin{array}{lcl} (4, 7) & y = -\frac{1}{2}x + c \\ x \ y & 7 = -\frac{1}{2}(4) + c \\ & 7 = -2 + c \\ & c = 9 \end{array}$$

$$y = -\frac{1}{2}x + 9 \quad (3)$$

3.

A straight line passes through the points (0, 5) and (3, 17).

Find the equation of the straight line. $x_1 \ y_1 \quad x_2 \ y_2$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{17 - 5}{3 - 0} \\ &= \frac{12}{3} \\ &= 4 \end{aligned}$$

$$\begin{array}{lcl} (0, 5) & y = 4x + c \\ x \ y & 5 = 4(0) + c \\ & c = 5 \end{array}$$

$$y = 4x + 5 \quad (3)$$

4. Show that line $3y = 4x - 14$ is perpendicular to line $4y = -3x + 48$.

$$3y = 4x - 14 \quad (\div 3)$$

$$y = \frac{4}{3}x - \frac{14}{3}$$

$$m = \frac{4}{3}$$

$$4y = -3x + 48 \quad (\div 4)$$

$$y = -\frac{3}{4}x + 12$$

$$m = -\frac{3}{4}$$

$$\frac{4}{3} \times -\frac{3}{4} = -1$$

..... (4)

5. Here are the equations of 5 straight lines.

$$P: y = 2x + 5$$

$$Q: y = -2x + 5$$

$$R: y = x + 5$$

$$S: y = -\frac{1}{2}x + 6$$

$$T: y = \frac{1}{2}x + 1$$

- a) Write down the letter of the line that is parallel to $y = x + 6$

.....^R..... (1)

- b) Write down the letter of the line that is perpendicular to $y = 2x - 1$

.....^S..... (1)

6. The point A has the coordinates (2,5)
The point B has the coordinates (6,7)

a) Find the mid point of AB

$$(4, 6) \dots (2)$$

b) Find the gradient of the line that passes through AB

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \begin{matrix} (2, 5) & (6, 7) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

$$= \frac{7 - 5}{6 - 2}$$

$$= \frac{2}{4} = \frac{1}{2}$$

$$\dots \frac{1}{2} \dots (2)$$

c) Find the equation of the perpendicular bisector to AB

$$\text{perpendicular gradient} = -2$$

$$y = -2x + c \quad \begin{matrix} (4, 6) \\ x & y \end{matrix}$$

$$6 = -2(4) + c$$

$$6 = -8 + c$$

$$c = 14$$

$$y = -2x + 14 \dots (3)$$

7. A circle C has centre (x_1, y_1) (2,5)
The point A (x_2, y_2) (11, 8) lies on the circumference of the circle

Find the equation of the tangent to the circle at A

Gradient of radius: $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{8 - 5}{11 - 2}$$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

perpendicular gradient = -3

$$y = -3x + c \quad \begin{matrix} (11, 8) \\ x \quad y \end{matrix}$$

$$8 = -3(11) + c$$

$$8 = -33 + c$$

$$c = 41$$

$$y = -3x + 41 \quad \text{.....} \quad (5)$$

8. A circle has the equation $x^2 + y^2 = 5$

a) Write down the centre of the circle

$$(0, 0) \dots (1)$$

b) Write down the exact length of the radius of the circle

$$\sqrt{5} \dots (1)$$

P is the point (1, 2) on the circle $x^2 + y^2 = 5$

c) Work out the equation of the tangent to the circle at P

$$\begin{array}{cc} (0, 0) & (1, 2) \\ x_1, y_1 & x_2, y_2 \end{array} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - 0}{1 - 0}$$
$$= 2$$

perpendicular gradient = $-\frac{1}{2}$

$$y = -\frac{1}{2}x + c \quad \begin{array}{c} (1, 2) \\ x \quad y \end{array}$$

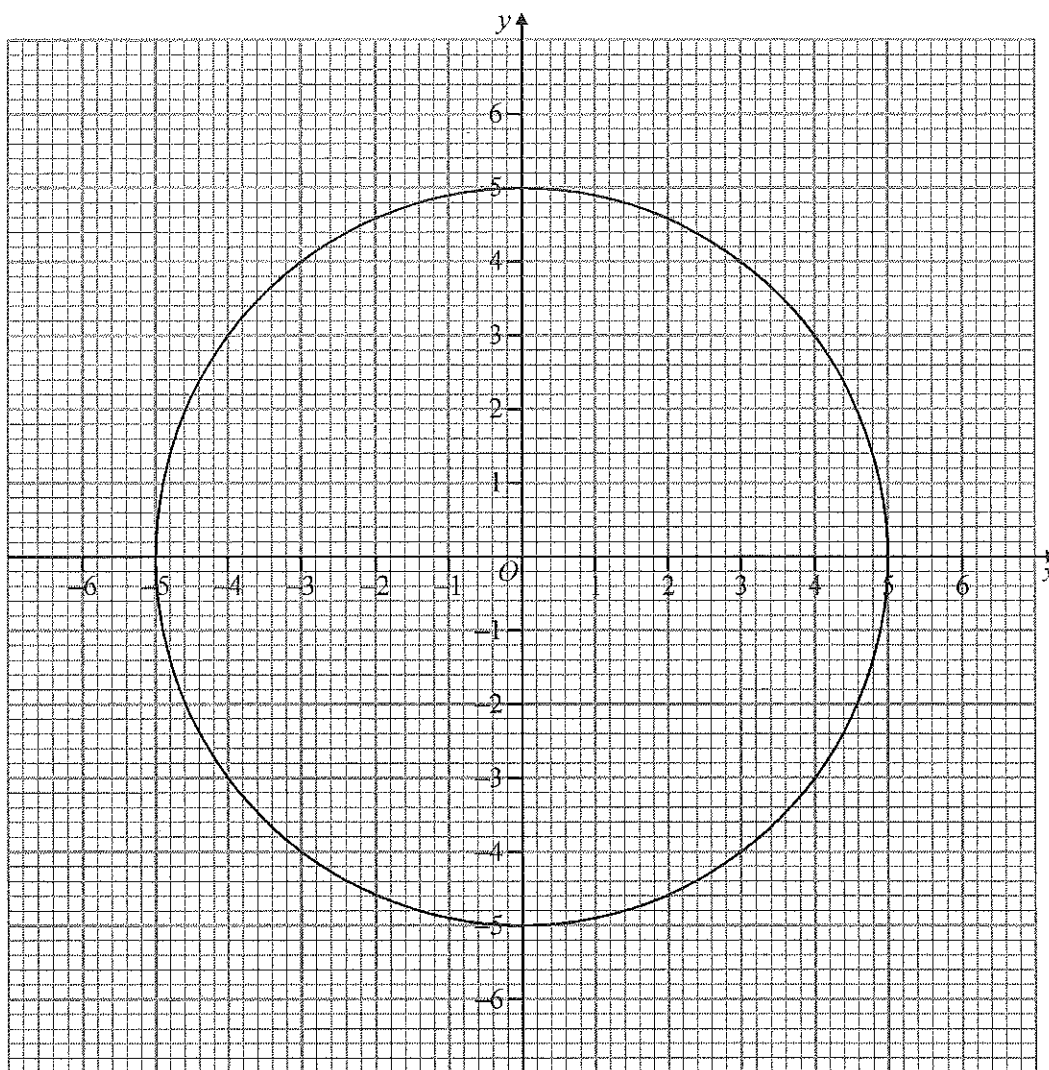
$$2 = -\frac{1}{2}(1) + c$$

$$2 = -\frac{1}{2} + c$$

$$2\frac{1}{2} = c$$

$$y = -\frac{1}{2}x + \frac{5}{2} \dots (4)$$

9. The diagram shows a circle of radius 5 cm, centre the origin.



Find the equation of the tangent to the circle at $(3,4)$
 $\begin{matrix} x & y \end{matrix}$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 0}{3 - 0} \\ &= \frac{4}{3} \end{aligned}$$

perpendicular gradient = $-\frac{3}{4}$

$$y = -\frac{3}{4}x + c$$

$$4 = -\frac{3}{4}(3) + c$$

$$4 = -\frac{9}{4} + c$$

$$c = \frac{25}{4}$$

$$y = -\frac{3}{4}x + \frac{25}{4} \quad (5)$$

$$\boxed{4 = \frac{16}{4}}$$