12 Constructions and Loci

12.1 Recap: Angles and Scale Drawing

The concepts in this unit rely heavily on knowledge acquired previously, particularly for angles and scale drawings, so in this first section we revise these two topics.

Example 1

In the diagram opposite, determine the size of each of the unknown angles.

Solution

Since

$c + 100° = 180°$ (BCD is a straight line)

$c = 180° - 100°$

$c = 80°$

(BCD is a straight line.)

Also, $b = c$, since the triangle is isosceles, so $b = 80°$.

Finally, since

$a + b + c = 180°$ (angles in a triangle add up to $180°$)

then

$a = 180° - (80° + 80°)$

so $a = 20°$
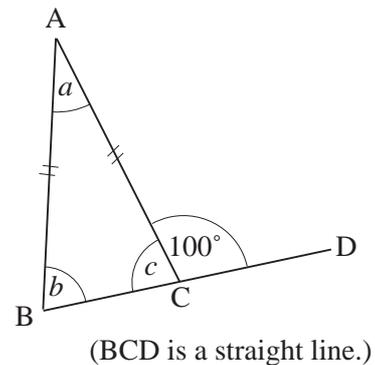
Example 2

In the diagram opposite, given that $a = 65°$, determine the size of each of the unknown angles.

Solution

$b = 180° - a$ (angles on a straight line are supplementary, i.e. they add up to $180°$)

$b = 180° - 65°$

$b = 115°$

$c = a = 65°$ (vertically opposite angles)

$d = b = 115°$ (corresponding angles, as the lines are parallel)

$e = a = 65°$ (corresponding angles)

$f = a = 65°$ (alternate angles)
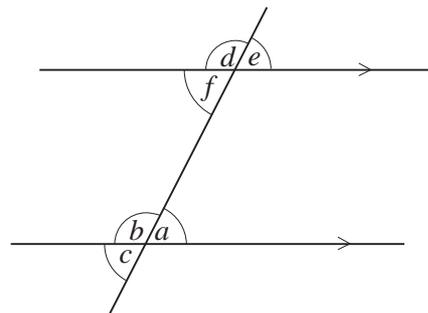
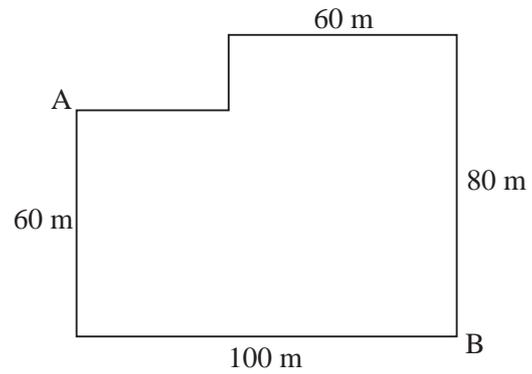## Example 3

Draw an accurate plan of the car park which is sketched here. Use the scale
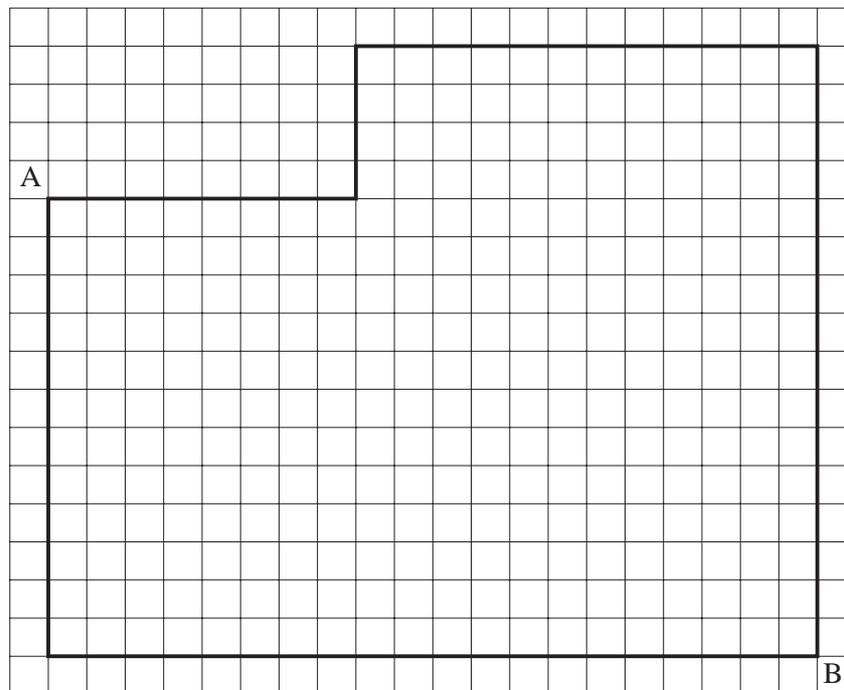
    1 cm ≡ 10 m.

Estimate the distance AB.

Solution

The equivalent lengths are:

    100 m ≡ 10 cm,   80 m ≡ 8 cm,   60 m ≡ 6 cm,

giving the following scale drawing:

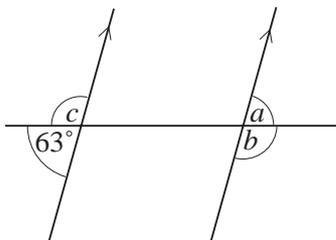
In the scale drawing, AB = 11.7 cm, which gives an actual distance AB = 117 m in the car park.
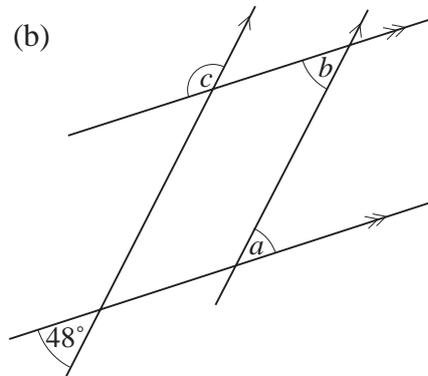
Exercises

1. Determine the size of each of the angles marked with a letter in the following diagrams, giving reasons for your answers.
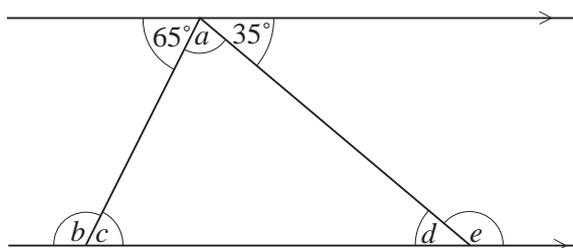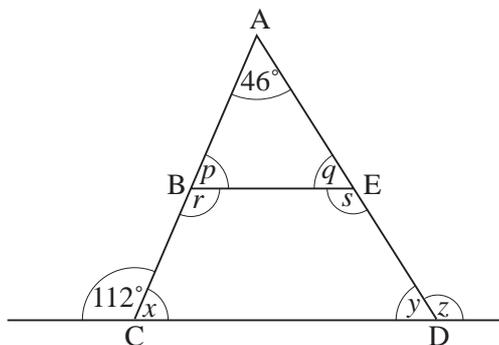
    (a)

    (b)

2. Determine the size of each of the angles marked with a letter in the following diagram:
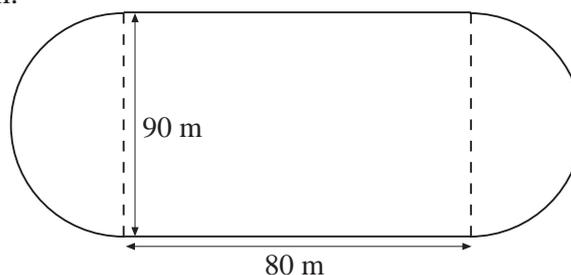
3. BCDE is a trapezium. Determine the size of each of the angles marked with a letter in the diagram, giving reasons for your answers.
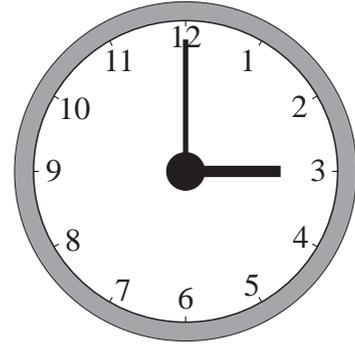
4. Draw a scale drawing of the running track shown in the sketch below. The radius of the semicircles is 45 m.

    Use a scale of

    $1 \text{ cm} \equiv 10 \text{ m}.$

    90 m

    80 m

5. (a) The time on this clock is 3 o'clock.

What is the *size* of the *angle* between the hands?

(b) Write down the whole number missing from this sentence:

At .... o'clock the size of the angle between the hands is 180 °.

(c) What is the size of the *angle* between the hands at 1 o'clock?

(d) What is the size of the *angle* between the hands at 5 o'clock?

(e) How long does it take for the *minute* hand to move 360 ° ?

(KS3/99/Ma/Tier 3-5/P2)

6. (a) Which *two* of these angles are the *same size*?
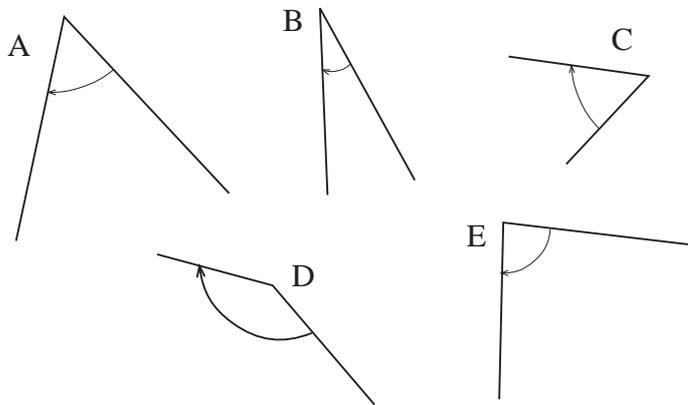
(b) Draw an angle which is *bigger* than a *right angle*.

(c) Kelly is facing *North*.
She turns *clockwise* through 2 right angles.
Which direction is she facing now?

(d) Aled is facing *West*.
He turns *clockwise* through 3 right angles.
Which direction is he facing now?
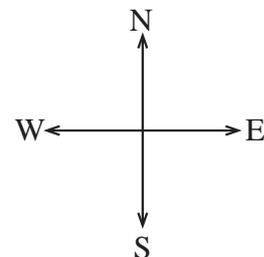
(KS3/98/Ma/Tier 3-5/P1)

7. The shape below has 3 *identical white* tiles and 3 *identical grey* tiles.

   The sides of each tile are all the same length.

   Opposite sides of each tile are parallel.

   One of the angles is $70\,°$.
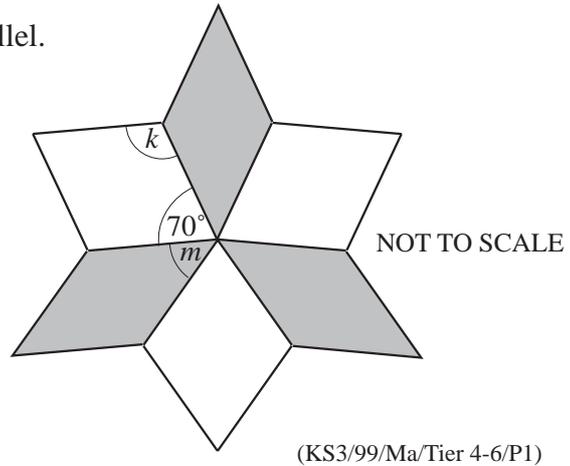
   (a) Calculate the size of *angle k*.

   (b) Calculate the size of *angle m*.
   Show your working.

   NOT TO SCALE

   (KS3/99/Ma/Tier 4-6/P1)

8. Kay is drawing shapes on her computer.

   (a) She wants to draw this triangle. She needs to know angles *a, b* and *c*.

   Calculate angles *a, b* and *c*.

   NOT TO SCALE
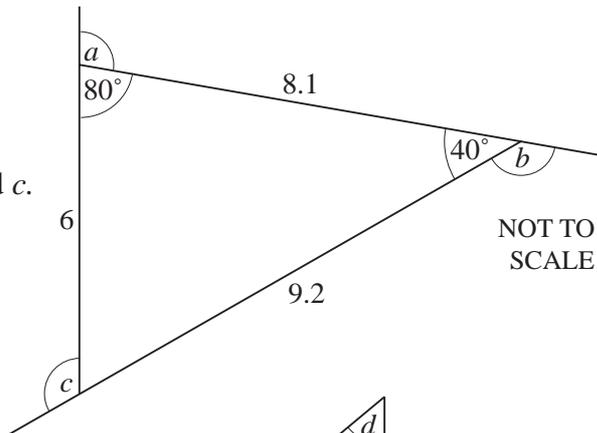
   (b) Kay draws a rhombus:

   Calculate angles *d* and *e*.

   NOT TO SCALE
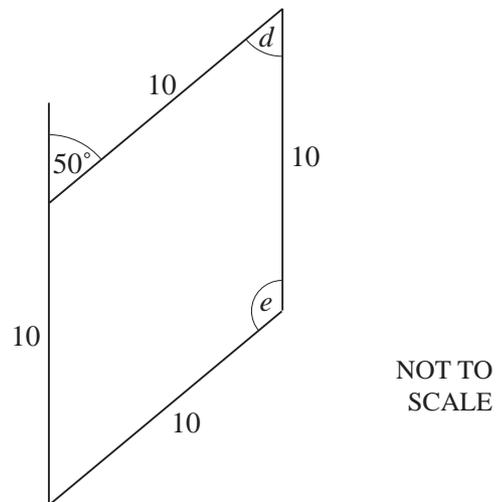
   (c) Kay types the instructions to draw a regular pentagon:

   repeat 5 [forward 10, left turn 72]

   Complete the following instructions to draw a regular hexagon:

   repeat 6 [forward 10, left turn ....]

   (KS3/97/Ma/Tier 4-6/P1)

9. In the scale drawing, the shaded area represents a lawn.

   There is a wire fence *all around* the lawn.

   The shortest distance from the fence to the edge of the lawn is *always* 6 m.

   On a copy of the diagram, draw *accurately* the position of the fence.

Scale: 1 cm to 3 m

3 m

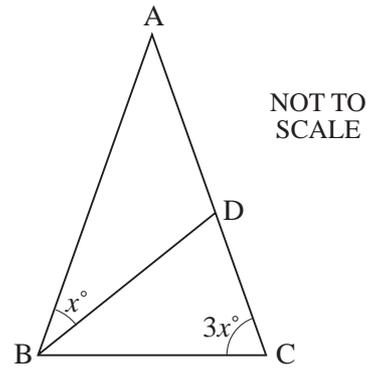3 m

Lawn
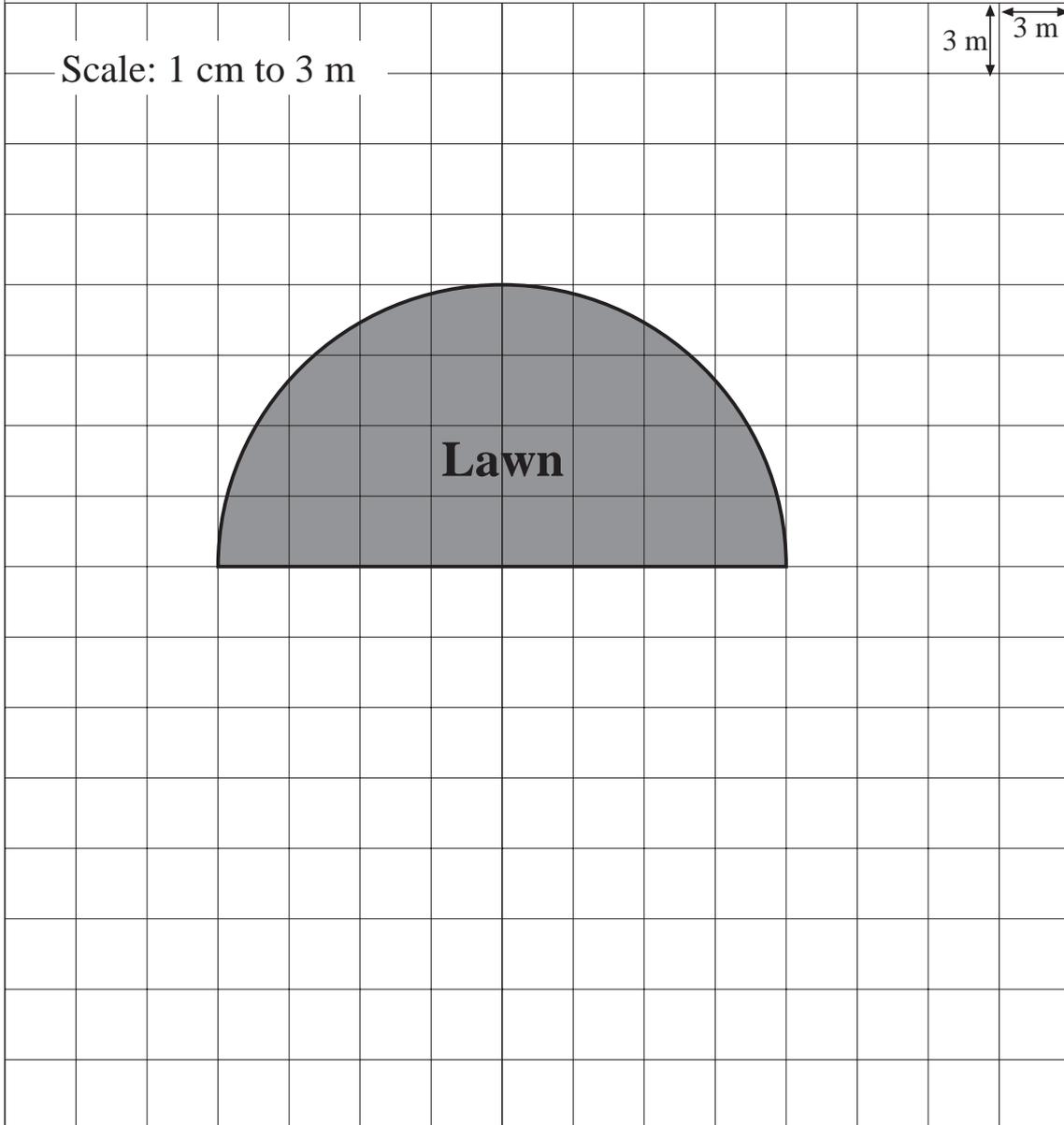
(KS3/98/Ma/Tier 6-8/P1)

10. Look at the diagram:

Side AB is the same length as side AC.

Side BD is the same length as side BC.

Calculate the value of *x*.

Show your working.

NOT TO
SCALE

(KS3/99/Ma/Tier 6-8/P1)

12.2 Constructions

In this section we look at how to construct triangles and various lines. You will
need a ruler, a protractor and a pair of compasses to be able to draw these
constructions. The following examples illustrate some of the techniques that you
will need to use.

Example 1

Construct the perpendicular bisector
of the line AB.

Then label the midpoint of AB, M.
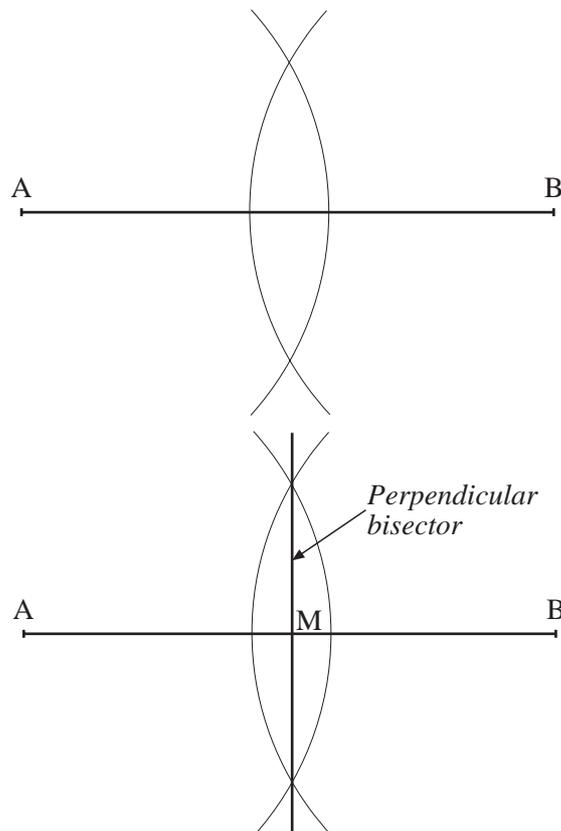
A ————————————————————— B

Solution

There are many lines that cut AB
exactly in half. We have to
construct the one that is
perpendicular to AB.

We begin by drawing arcs of equal
radius, centred on the points A and
B, as shown in the diagram.
The radius of these arcs should be
roughly $\frac{2}{3}$ to $\frac{3}{4}$ of the length AB.

Then draw a line through the
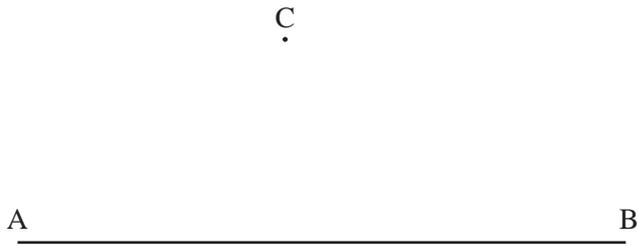intersection points of the two
arcs.

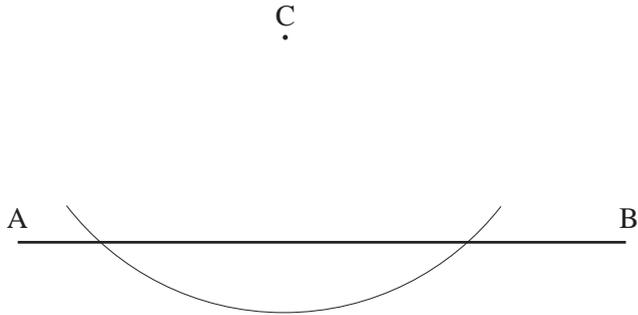The point where the bisector
intersects AB can then be
labelled M.

*Perpendicular
bisector*

Example 2

The diagram shows the line
AB and the point C.

Draw a line through C that
is perpendicular to AB.

C

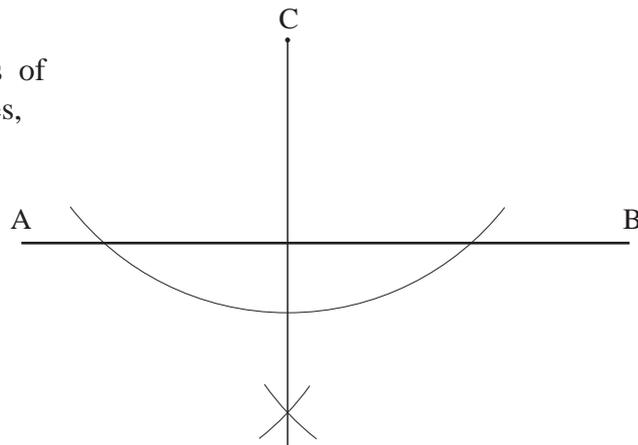A   B

Solution

Using C as the centre, draw
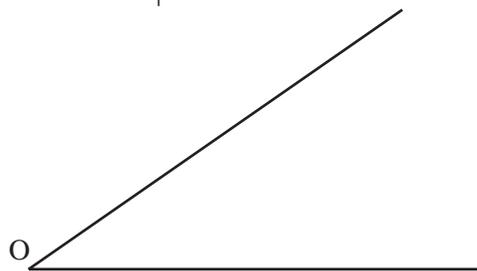an arc as shown.

C

A   B

Then using the intersection points of
this arc with the line AB as centres,
draw two further arcs with
radii of equal length. The
perpendicular line can then
be drawn from C through
the point where these
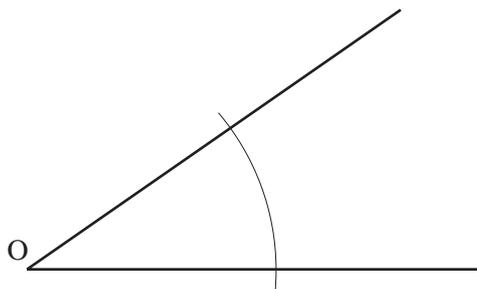two new arcs cross.

C

A   B

Example 3
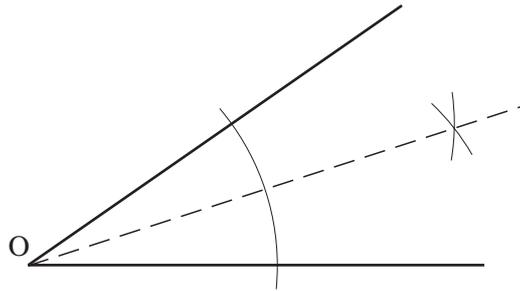
Bisect this angle.

O

Solution

To bisect an angle you need to draw
a line that cuts the angle in half.

First draw an arc using O as the centre.

O

Then draw two further arcs of equal
radius, using the points where the arc
intersects the lines as the centres.

The bisector can then be drawn from
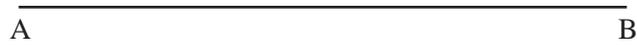O through the point where these
two new arcs cross.

Example 4

The triangle ABC is such that AB = 8 cm, $\angle$BAC = 40 $^\circ$ and $\angle$ABC = 60 $^\circ$.
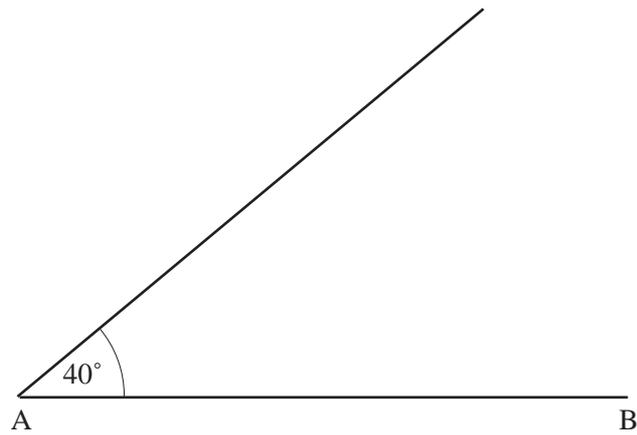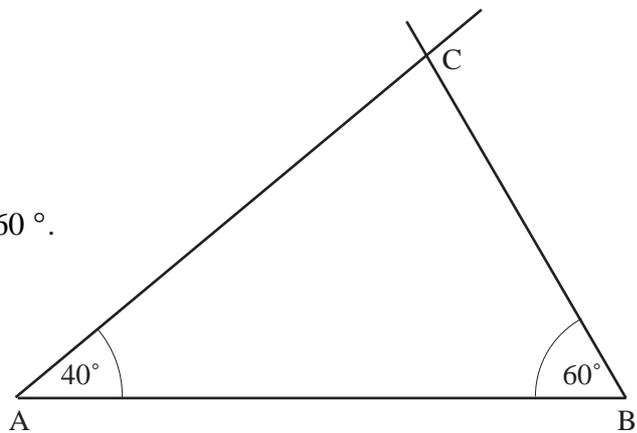Draw this triangle.

Solution

First draw the line AB of
length 8 cm.

At the left-hand-end of the line,
draw the $\angle$ BAC which is 40 $^\circ$.

Then draw the $\angle$ ABC which is 60 $^\circ$.

Exercises

1. (a) Draw a line of length 10 cm.

   (b) Construct the perpendicular bisector of the line.

   (c) Check that it *does* cut the line in half.

   (d) Use a protractor to check that it *is* perpendicular.

2. (a) Mark 3 points, not in a straight line, on a piece of paper and label them A, B and C. Draw a line from A to B.

   (b) Construct a line that is perpendicular to AB and passes through C.

   (c) Use a protractor to check that your line is perpendicular.

3. (a) Use a ruler and a protractor to construct the triangle ABC where
      AB = 6 cm, $\angle ABC = 60°$ and $\angle BAC = 50°$.

   (b) Construct a line that is perpendicular to AC and passes through the corner B.

4. (a) Draw a triangle with sides of length 7 cm, 4 cm and 6 cm.

   (b) Construct the perpendicular bisector of each side. What do you notice?

   (c) Draw a circle with its centre at the point where the lines intersect and that passes through each corner of the triangle.

   (d) Repeat this process for any other triangle. Does it still work?

5. (a) Draw the triangle which has sides of length 8 cm, 7 cm and 6 cm.

   (b) Construct the bisector of each angle of the triangle.

   (c) Using the point where the lines intersect as its centre, draw the largest circle that will fit inside the triangle.

6. The diagram shows how Ishmael constructed a 60° angle.

   (a) Construct a 60° angle in this way and then check that it *is* 60°.

   (b) Bisect your angle to obtain a 30° angle.

   (c) Construct the following angles, using a pair of compasses and a ruler.

(i) 120°	(ii) 240°	(iii) 300°
(iv) 90°	(v) 270°	(vi) 45°
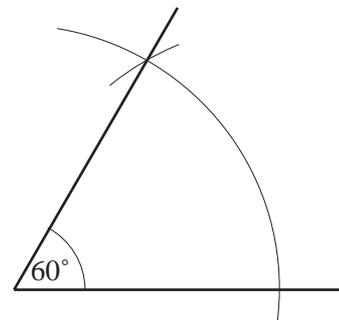
7. The triangle ABC is such that AB = 6 cm, AC = 7 cm and ∠BAC = 50 °.

    (a) Draw the triangle.

    (b) What is the length of the side BC ?

    (c) Construct a line that passes through C and is perpendicular to AB.

    (d) Hence calculate the area of the triangle.

8. A triangle PQR has PR = 6 cm, QR = 5 cm and ∠ QPR = 45 °. Abigail and Kirsty are asked to draw this triangle. They draw the two triangles below.
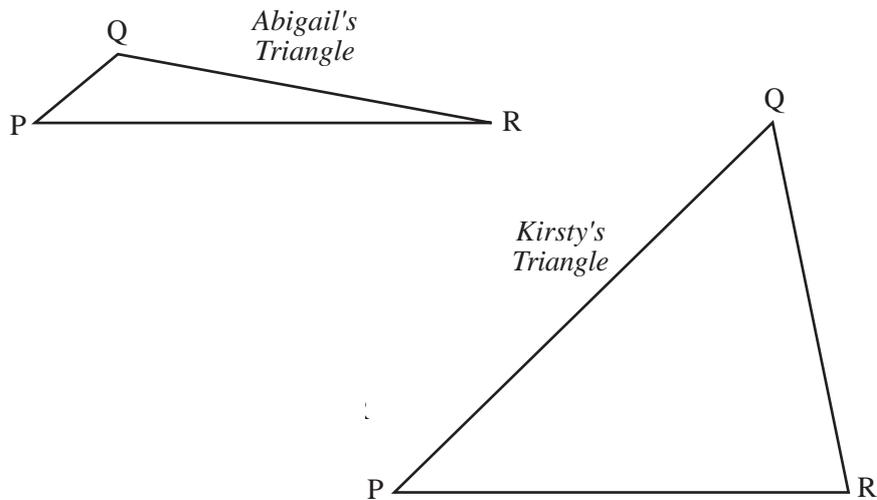
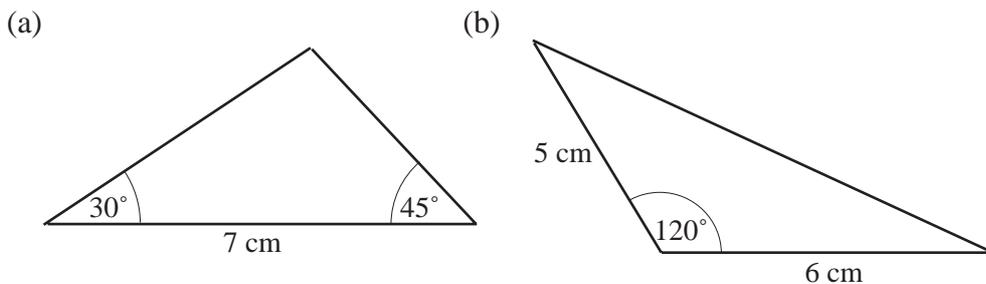   (a) Are they both correct?

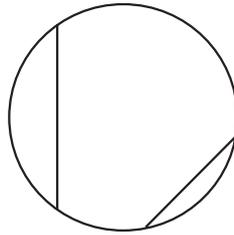   (b) Draw the two possible triangles ABC, given the information below.

    $$AB \quad = \quad 8 \text{ cm}$$

    $$BC \quad = \quad 7 \text{ cm}$$

    $$\angle BAC \quad = \quad 50 °$$

9. Construct each of the following triangles, without using a protractor.

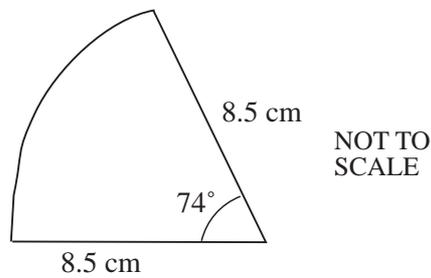   (a)   (b)

10.   Draw a circle and two chords like those shown in the diagram.

Construct the perpendicular bisector for each chord. What do you notice?
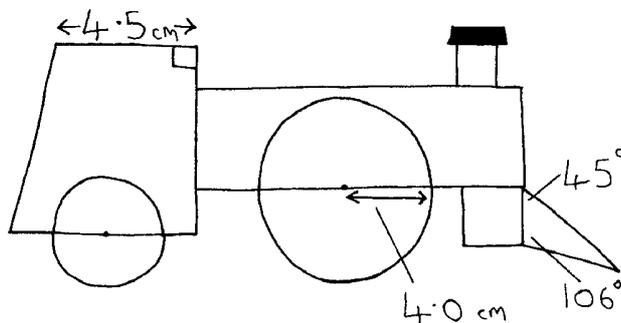
Do you think this will always be true?

11.   Here is a *rough sketch* of a sector of a circle.

8.5 cm

NOT TO
SCALE

74°

8.5 cm

Make an *accurate, full size* drawing of this sector.
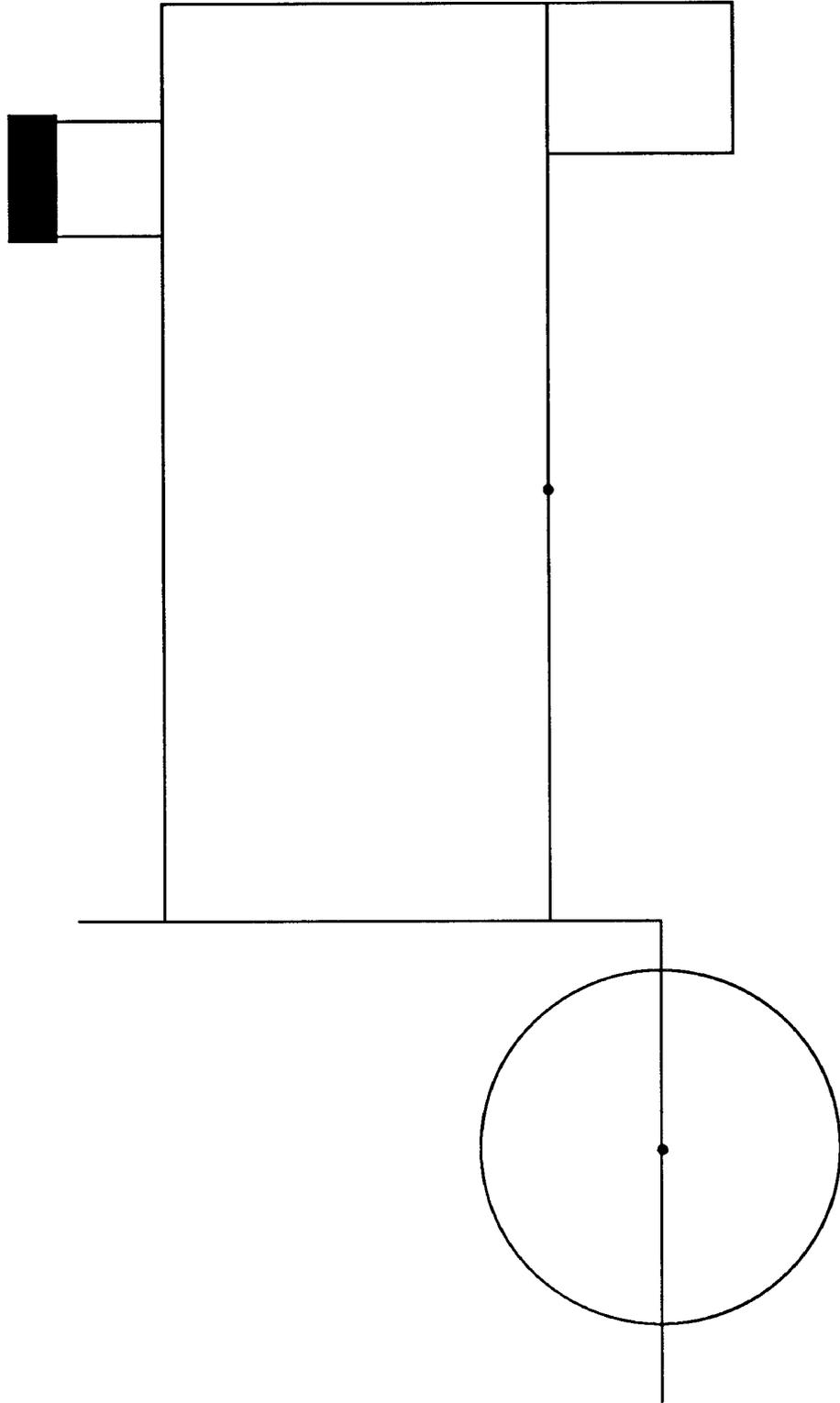
(KS3/97/Ma/Tier 5-7/P2)

12.   Jane wants to design a toy engine.

She makes a rough sketch to show some of the measurements.

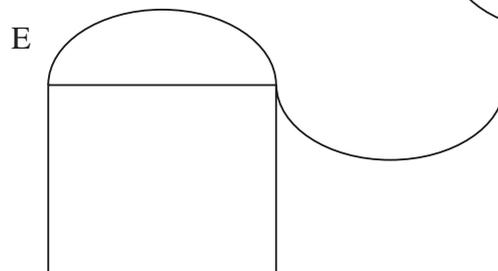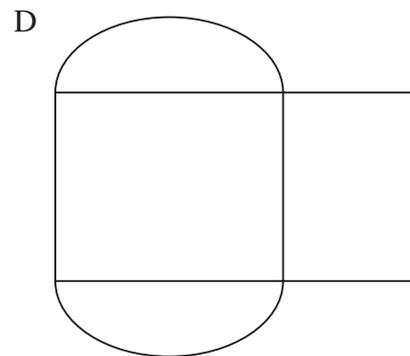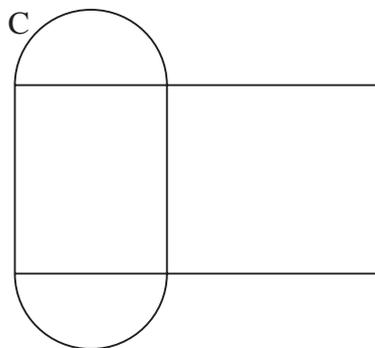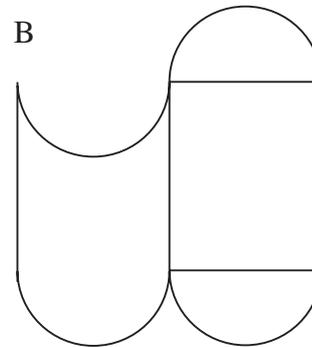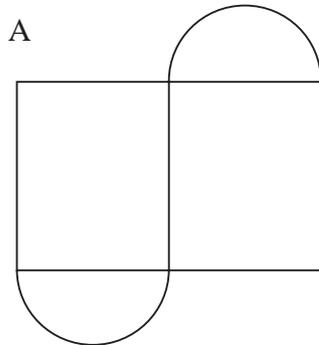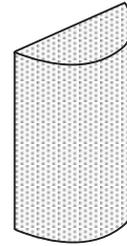Jane starts to draw the accurate side view.

On a copy of the following diagram, finish Jane's side view.

You will need a ruler, an angle measurer or protractor, and a pair of compasses.

(KS3/96/Ma/Tier 5-7/P1)

13. (a)   The top and the base of this box are *semi-circles.*

          Which *one* of the nets below could fold up to
          make a box like this?

     A

     B

     C

     D

     E

     (b)   This is a rough sketch of the *base* of a box.

          It is a *semi-circle,* with *diameter* 8 cm.

          Make an accurate, full size drawing
          of the *base* of the box.   ◄——— 8 cm ———►

          You will need a ruler and a pair of compasses.
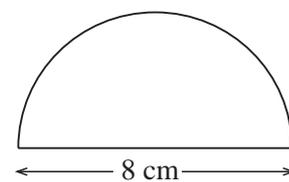
                                                    (KS3/98/Ma/Tier 3-5/P2)

12.3 Loci

A locus is a set of points all of which share some common property. A locus may be a point, a line, a curve or a region. The important point is that all the points that make up the locus have to satisfy the same rule or condition. For example, you might be asked to draw the locus of points that are a certain distance from a given point or line.
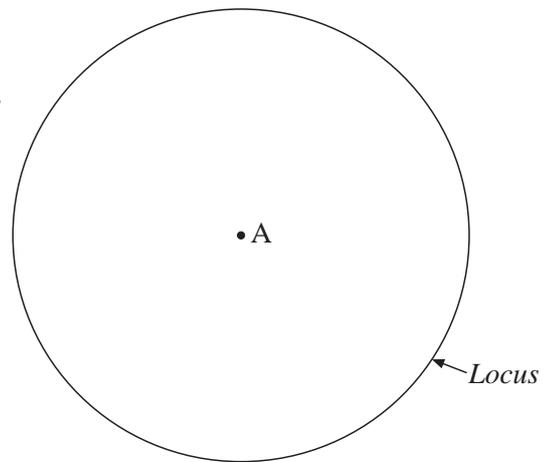
Example 1

Draw the locus of the points that are 3 cm from the point A.

Solution

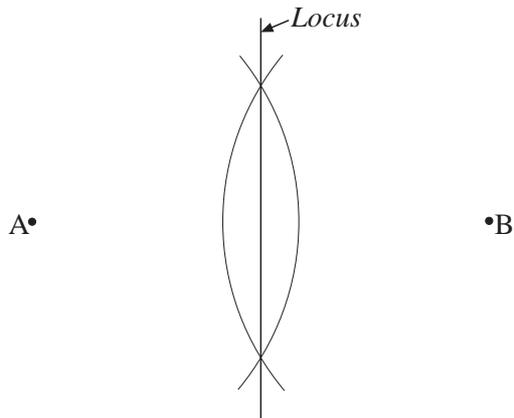The locus will simply be a circle, centre A, with radius 3 cm. Every point on the circle will be 3 cm from A.

• A

Locus

Example 2

Draw the locus of the points that are equidistant from A and B.
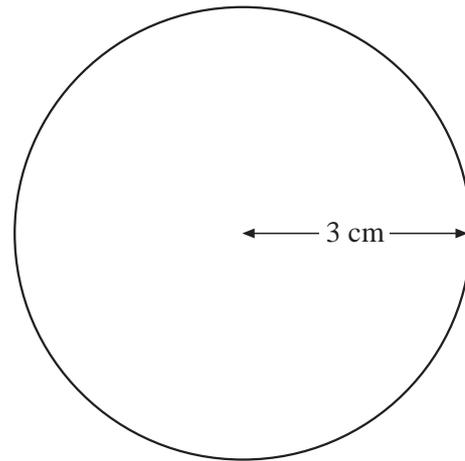
A•   •B

Solution

All the points must be the same distance from A as from B. The locus is the perpendicular bisector of the line AB.
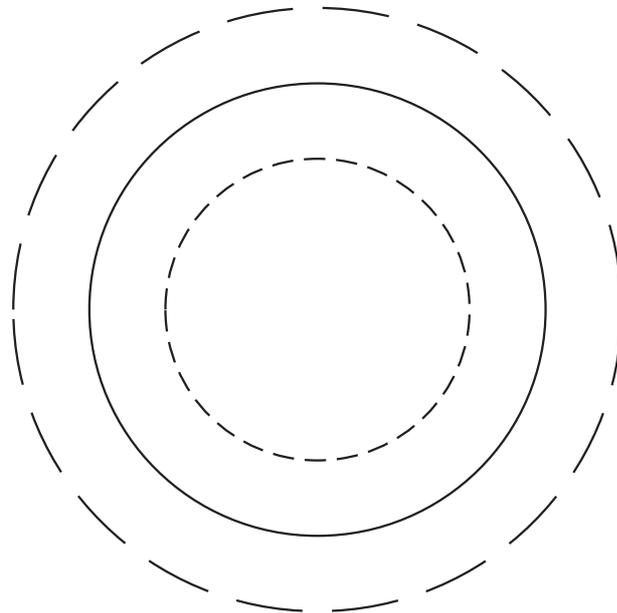
Locus

A•   •B

Example 3

Draw the locus of points that are 1 cm from this circle.

← 3 cm →

Solution

The locus is made up of 2 parts. 1 part consists of the points that are 1 cm from the circle and inside it; the other is those points that are 1 cm from the circle and are outside it.

Exercises

1. (a) Draw a line of length 5 cm.

   (b) Draw the locus of points that are 1 cm from the line.

2. (a) Draw a circle of radius 2 cm.

   (b) Draw the locus of points that are 2 cm from the circle.

   (c) On your diagram, shade the locus of points that are *less* than 2 cm from the circle.

3. (a) Draw the rectangle shown in the diagram.

   (b) Draw the locus of the points that are 1 cm from the rectangle.

   (c) Repeat part (b) for a rectangle that is 6 cm long and 5 cm wide.

4 cm

1 cm
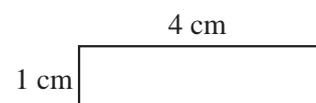
4. Construct the locus of the points that are equidistant from the two lines shown in the diagram.
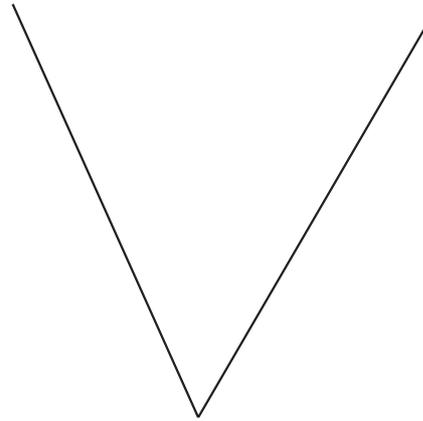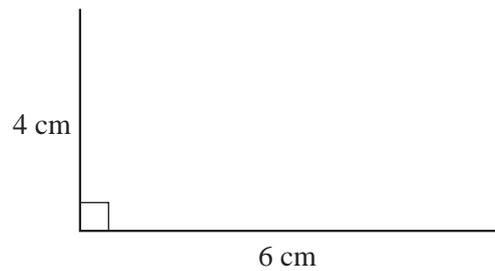
5. (a) Construct the triangle shown in the diagram.

   (b) Draw the locus of the points that are 1 cm from the triangle.

4 cm

3 cm

6 cm
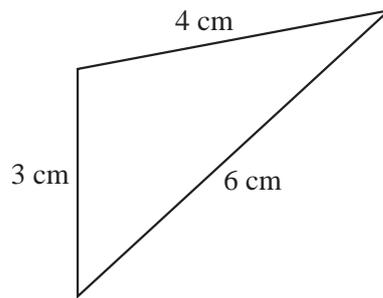
6. Draw the locus of the points that are 1 cm from the shape in the diagram.

4 cm

6 cm

7. Two points A and B are 6 cm apart.

    (a) Draw the locus of the points that are equidistant from A and B.

    (b) Draw the locus of points that are 5 cm from B.

    (c) Indicate the points that are 5 cm from A and B.

8. The points A and B are 9 cm apart. Draw the locus of the points that are twice as far from A as they are from B.

9. (a) Construct the triangle shown in the diagram.

    (b) Draw the locus of points that are equidistant from A and B and within 3 cm of C.
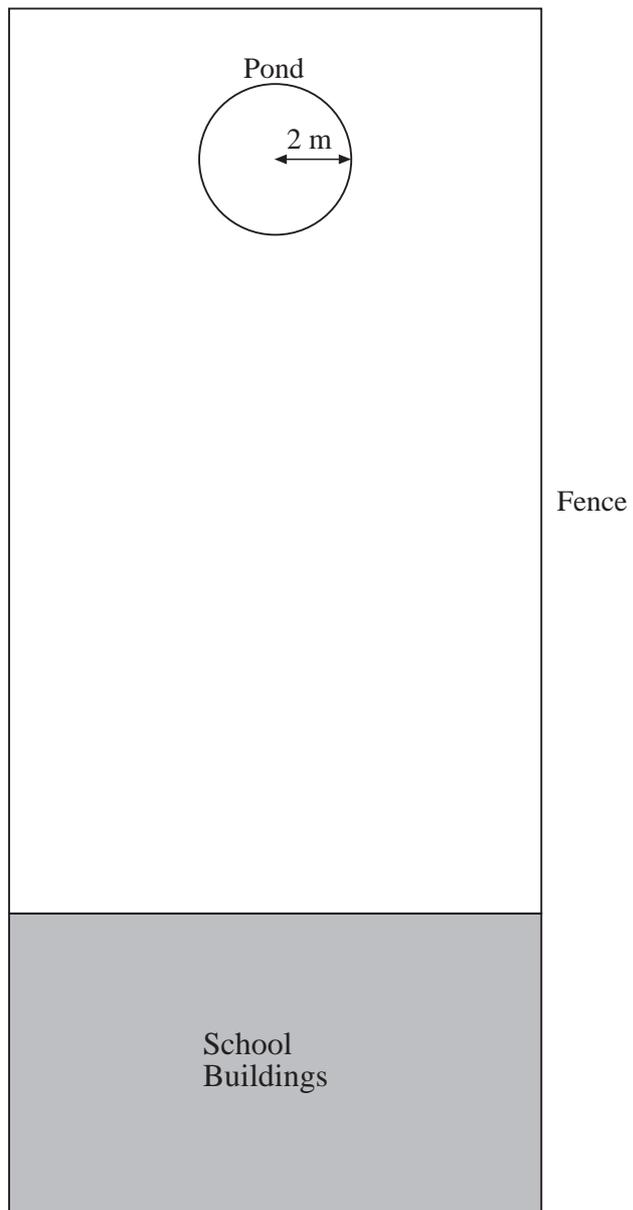
A

4 cm

6 cm

B

4 cm

C
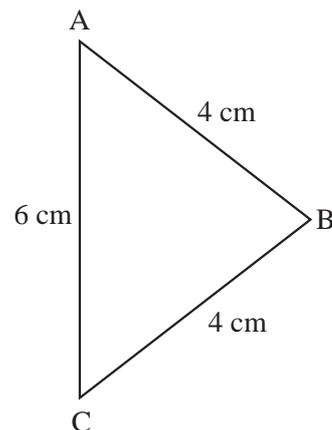
10.   A ladder has length 4 m. It initially leans against a vertical wall with its base on horizontal ground.

   The ladder slides down until it is lying horizontal on the ground.

   Draw the locus of the midpoint of the ladder, using a suitable scale drawing.

11.   Some pupils want to plant a tree in the school's garden.

   The tree must be at least 12 m from the school building.

   It must also be at least 10 m from the centre of the round pond.

   (a)   Show accurately on a copy of the following plan the *region* in which the tree can be planted.

   Shade in this region.

   SCALE   1 cm to 2 m

   Pond

   2 m

   (b)   The pupils want to make a gravel path of width 1 m around the pond. Calculate the *area* of the path.

   Show your working.

   Fence

   School
   Buildings

(KS3/97/Ma/Tier 6-8/P2)