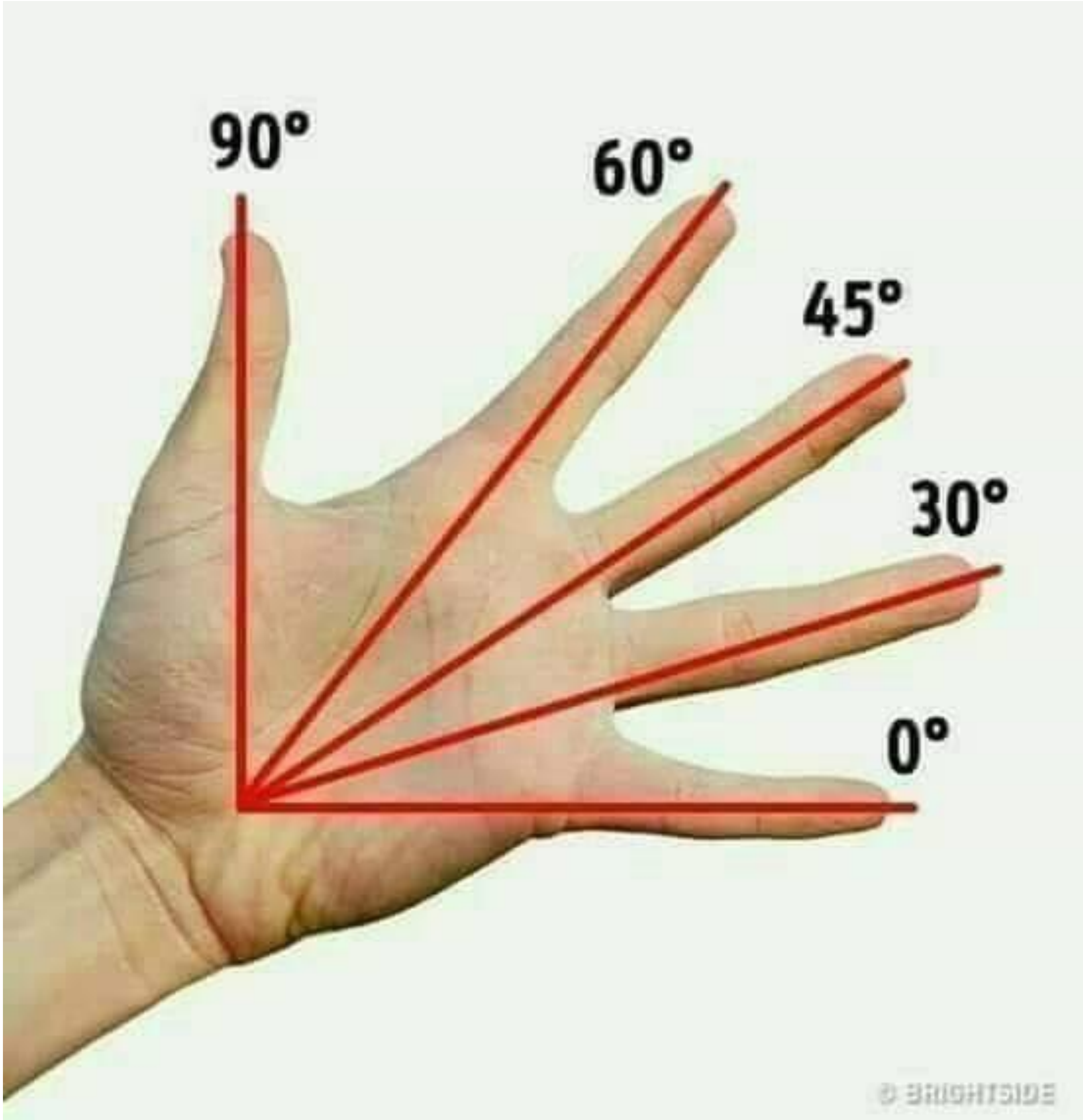
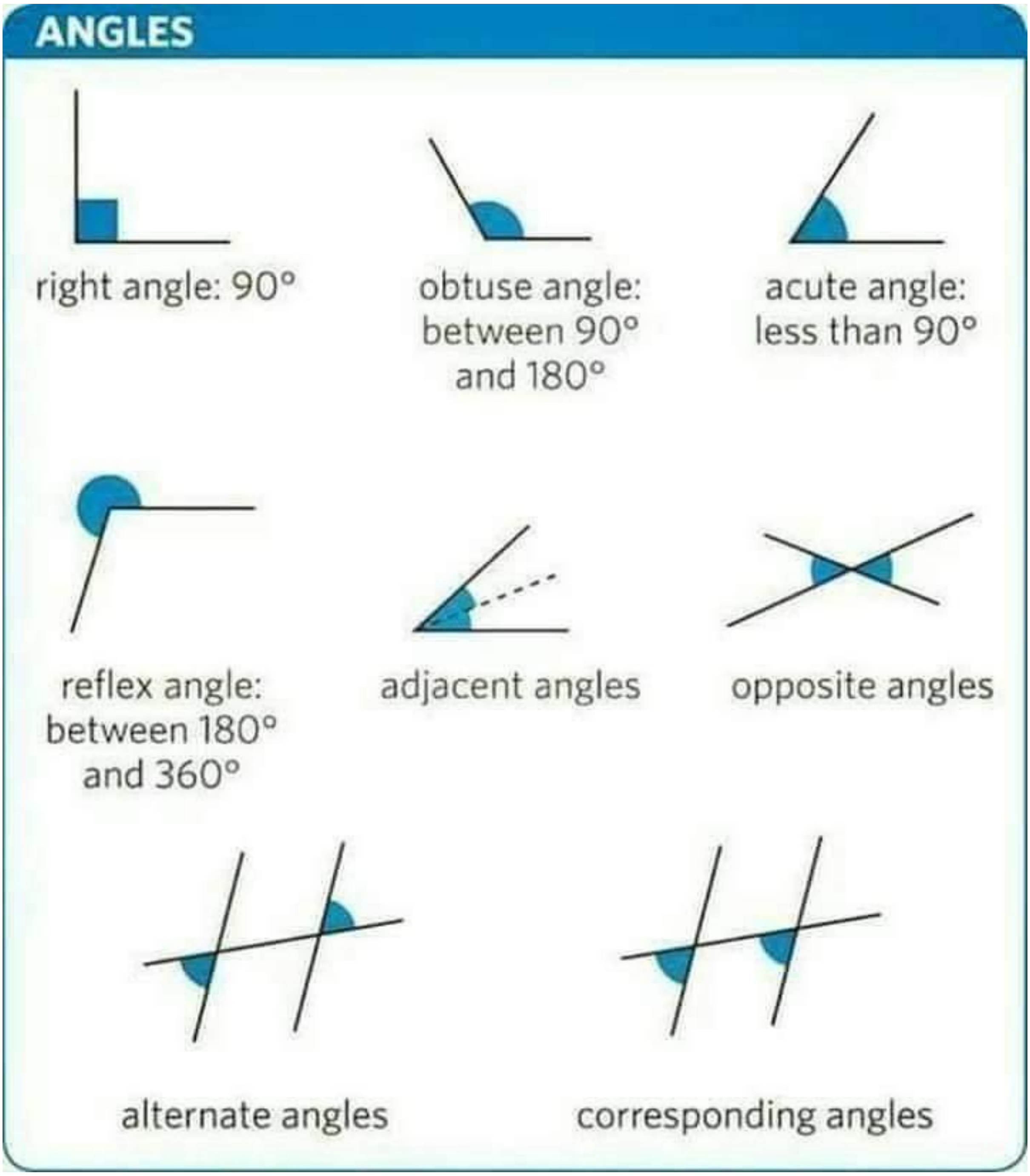
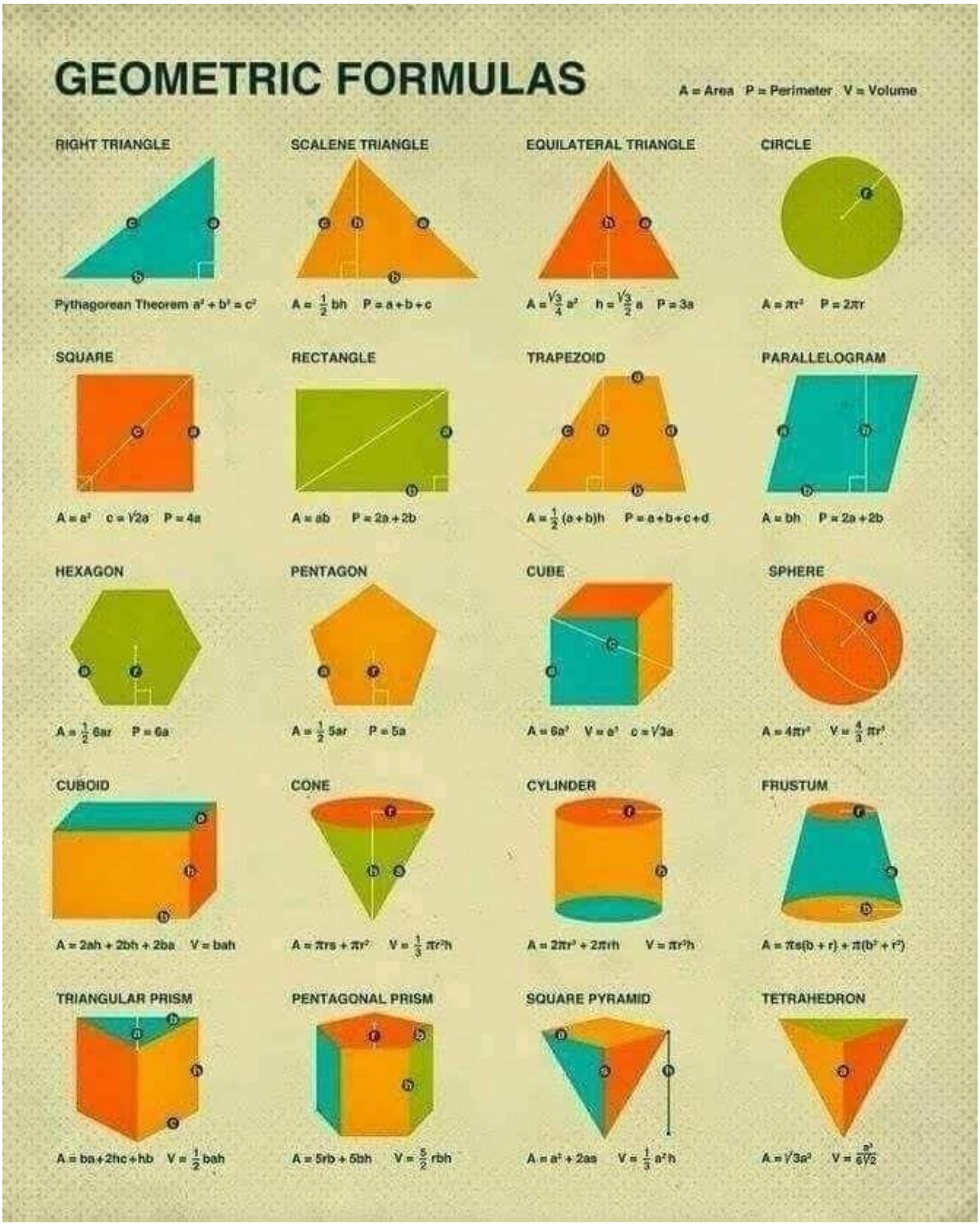
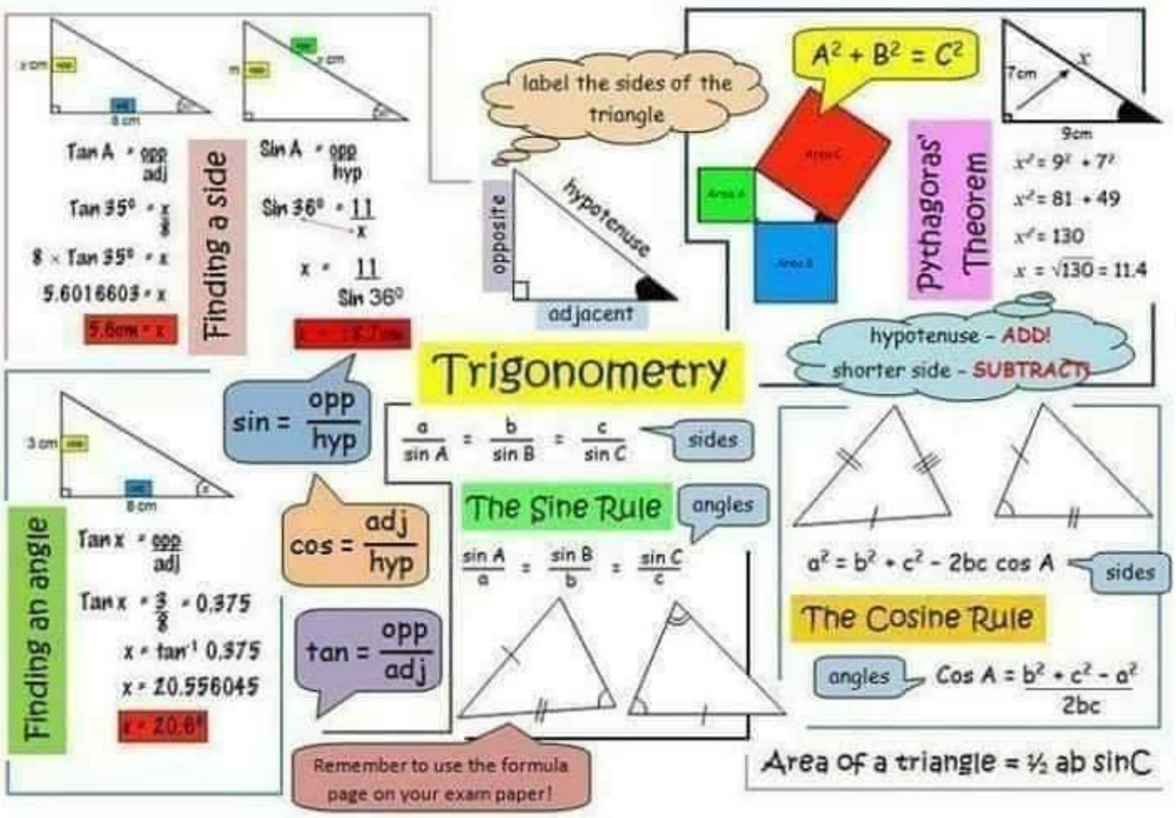
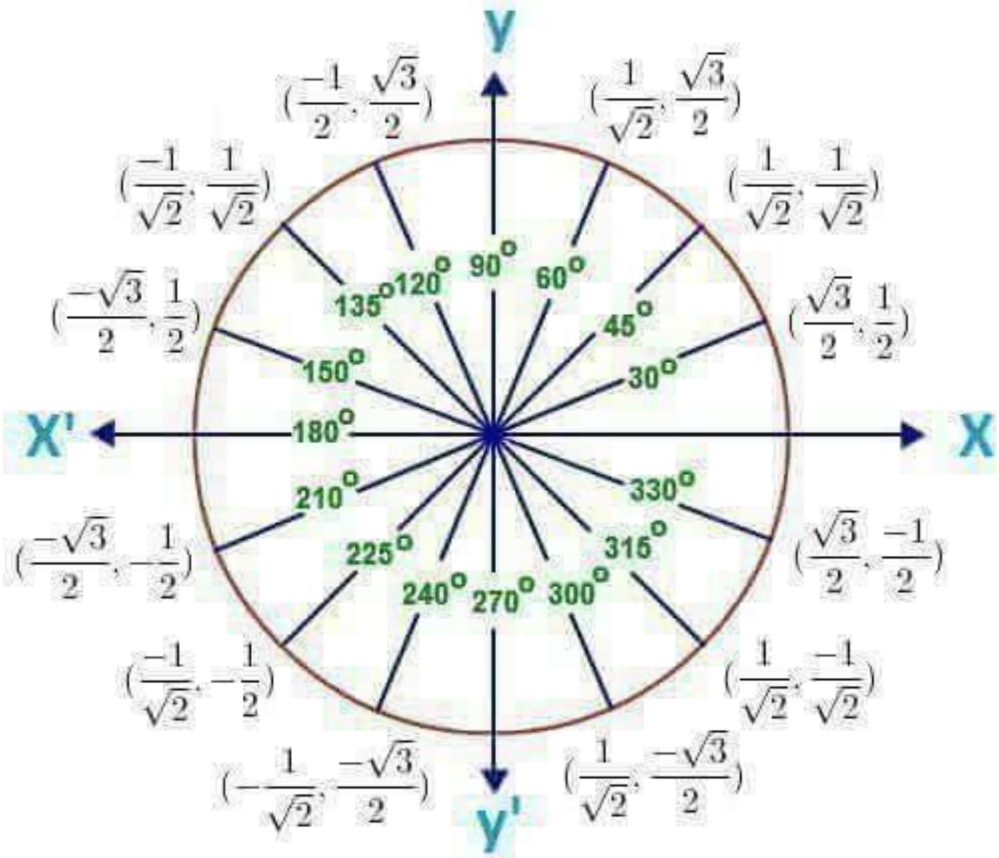

90°
60°
45°
30°
0°
© BRIGHTSIDE

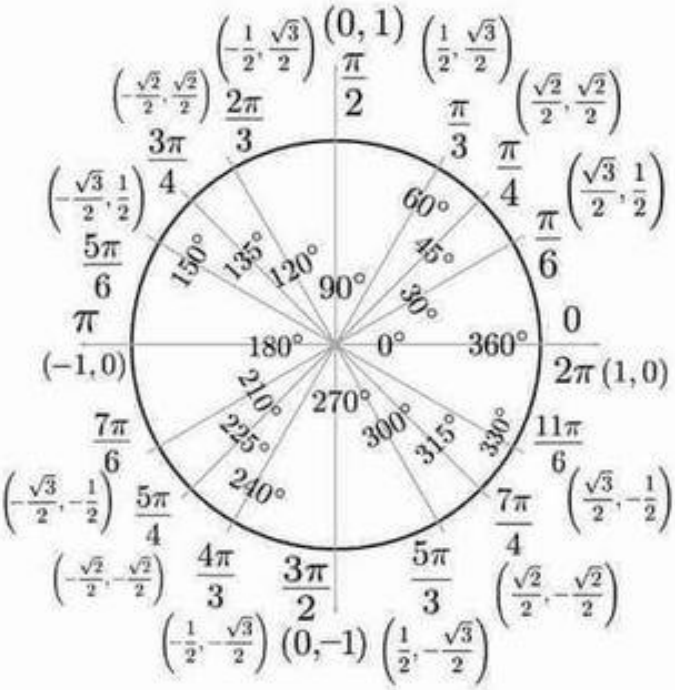

ANGLES
right angle: 90°
obtuse angle: between 90° and 180°
acute angle: less than 90°
reflex angle: between 180° and 360°
adjacent angles
opposite angles
alternate angles
corresponding angles


GEOMETRIC FORMULAS
A = Area P = Perimeter V = Volume
RIGHT TRIANGLE
Pythagorean Theorem a² + b² = c²
SCALENE TRIANGLE
A = 1/2 bh P = a + b + c
EQUILATERAL TRIANGLE
A = √3/4 a² h = √3/2 a P = 3a
CIRCLE
A = πr² P = 2πr
SQUARE
A = a² c = √2a P = 4a
RECTANGLE
A = ab P = 2a + 2b
TRAPEZOID
A = 1/2 (a + b)h P = a + b + c + d
PARALLELOGRAM
A = bh P = 2a + 2b
HEXAGON
A = 1/2 6ar P = 6a
PENTAGON
A = 1/2 5ar P = 5a
CUBE
A = 6a² V = a³ c = √3a
SPHERE
A = 4πr² V = 4/3 πr³
CUBOID
A = 2ah + 2bh + 2ba V = bah
CONE
A = πrs + πr² V = 1/3 πr²h
CYLINDER
A = 2πr² + 2πrh V = πr²h
FRUSTUM
A = πs(b + r) + π(b² + r²)
TRIANGULAR PRISM
A = ba + 2hc + hb V = 1/2 bah
PENTAGONAL PRISM
A = 5rb + 5bh V = 5/2 rbh
SQUARE PYRAMID
A = a² + 2as V = 1/3 a²h
TETRAHEDRON
A = √3a² V = a³/6√2


Trigonometry
Finding a side
Tan A = opp/adj
Tan 35° = x/8
8 × Tan 35° = x
5.6016603 = x
5.6cm = x
Sin A = opp/hyp
Sin 36° = 11/x
x = 11/Sin 36°
label the sides of the triangle
hypotenuse
opposite
adjacent
$A^2 + B^2 = C^2$
Pythagoras' Theorem
7cm
9cm
$x^2 = 9^2 + 7^2$
$x^2 = 81 + 49$
$x^2 = 130$
$x = \sqrt{130} = 11.4$
hypotenuse - ADD!
shorter side - SUBTRACT!
Finding an angle
3 cm
8 cm
Tan x = opp/adj
Tan x = 3/8 = 0.375
x = tan⁻¹ 0.375
x = 20.556045
x = 20.6°
sin = opp/hyp
cos = adj/hyp
tan = opp/adj
a/sin A = b/sin B = c/sin C
sides
The Sine Rule
angles
sin A/a = sin B/b = sin C/c
Remember to use the formula page on your exam paper!
$a^2 = b^2 + c^2 - 2bc \cos A$
sides
The Cosine Rule
angles
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
Area of a triangle = ½ ab sinC


y
y'
X
X'
90°
60°
45°
30°
120°
135°
150°
180°
210°
225°
240°
270°
300°
315°
330°
$(\frac{-1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2})$
$(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
$(\frac{-\sqrt{3}}{2}, \frac{1}{2})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
$(\frac{-\sqrt{3}}{2}, -\frac{1}{2})$
$(\frac{\sqrt{3}}{2}, \frac{-1}{2})$
$(\frac{-1}{\sqrt{2}}, -\frac{1}{2})$
$(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$
$(-\frac{1}{\sqrt{2}}, \frac{-\sqrt{3}}{2})$
$(\frac{1}{\sqrt{2}}, \frac{-\sqrt{3}}{2})$

Unit Circle



Unit Circle Table

Degree	cos	sin	tan	sec	csc	cot
0°	1	0	0	1	undefined	undefined
30°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	2	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	2	$\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
90°	0	1	undefined	undefined	1	0
120°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
135°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
150°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\sqrt{3}$
180°	-1	0	0	-1	undefined	undefined
210°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\sqrt{3}$
225°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
270°	0	-1	undefined	undefined	-1	0
300°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	2	$\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
315°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
330°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\sqrt{3}$
360°	1	0	0	1	undefined	undefined

Powers and Square Roots To Memorize!!!		
$1^2 = 1$	$1^3 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$2^3 = 8$	$\sqrt{4} = 2$
$3^2 = 9$	$3^3 = 27$	$\sqrt{9} = 3$
$4^2 = 16$	$4^3 = 64$	$\sqrt{16} = 4$
$5^2 = 25$	$5^3 = 125$	$\sqrt{25} = 5$
$6^2 = 36$	$6^3 = 216$	$\sqrt{36} = 6$
$7^2 = 49$	$1^4 = 1$	$\sqrt{49} = 7$
$8^2 = 64$	$2^4 = 16$	$\sqrt{64} = 8$
$9^2 = 81$	$3^4 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$4^4 = 256$	$\sqrt{100} = 10$
$11^2 = 121$	$5^4 = 625$	$\sqrt{121} = 11$
$12^2 = 144$	$1^5 = 1$	$\sqrt{144} = 12$
$13^2 = 169$	$2^5 = 32$	$\sqrt{169} = 13$
$14^2 = 196$	$3^5 = 243$	$\sqrt{196} = 14$
$15^2 = 225$	$4^5 = 1024$	$\sqrt{225} = 15$
$16^2 = 256$	$1^6 = 1$	$\sqrt{256} = 16$
$17^2 = 289$	$2^6 = 64$	$\sqrt{289} = 17$
$18^2 = 324$	$3^6 = 729$	$\sqrt{324} = 18$
$19^2 = 361$	$1^7 = 1$	$\sqrt{361} = 19$

Laws of Indices

www.cazoommaths.com

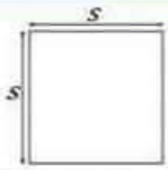
Regra	Exemplo
$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^8$
$a^m \div a^n = a^{m-n}$	$5^7 \div 5^3 = 5^4$
$(a^m)^n = a^{m \times n}$	$(10^3)^7 = 10^{21}$
$a^1 = a$	$17^1 = 17$
$a^0 = 1$	$34^0 = 1$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{6}\right)^2 = \frac{25}{36}$
$a^{-m} = \frac{1}{a^m}$	$9^{-2} = \frac{1}{81}$
$a^{\frac{x}{y}} = \sqrt[y]{a^x}$	$49^{\frac{1}{2}} = \sqrt[2]{49} = 7$

TRANSITION TO ALGEBRA FORMULA CHART

Distance formula $d = rt$	Percent proportion $\frac{is}{of} = \frac{\%}{100}$
Simple Interest formula $I = prt$	Percent of Change $\frac{difference}{original} = \frac{\%}{100}$
Distance between to ordered pairs $d = \sqrt{(x_2 - x)^2 + (y_2 - y_1)^2}$	Midpoint $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$
Pythagorean Theorem $c^2 = a^2 + b^2$	Slope of a line $m = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form $y = mx + b$	Perimeter of Square $P = 4s$
Perimeter of Rectangle $P = 2l + 2w$	Volume of Rectangular Prism $V = lwh$
Volume of Cube $V = s^3$	Area of Square $A = s^2$
Area of Rectangle $A = bh$	Area of Triangle $A = \frac{bh}{2}$
Area of Circle $A = \pi r^2$	Area of Trapezoid $A = \frac{1}{2}h(b_1 + b_2)$
Circumference of Circle $C = \pi d$	

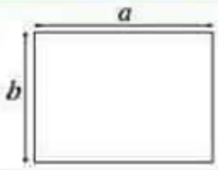
SQUARE

$P = 4s$
$A = s^2$



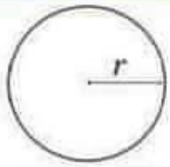
RECTANGLE

$P = 2a + 2b$
$A = ab$



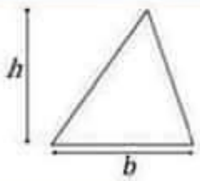
CIRCLE

$P = 2\pi r$
$A = \pi r^2$



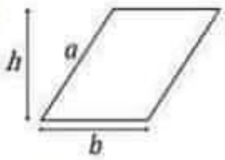
TRIANGLE

$P = a + b + c$
$A = \frac{1}{2}bh$



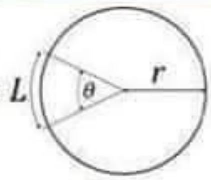
PARALLELOGRAM

$P = 2a + 2b$
$A = bh$



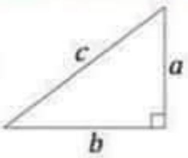
CIRCULAR SECTOR

$L = \pi r \frac{\theta}{180°}$
$A = \pi r^2 \frac{\theta}{360°}$



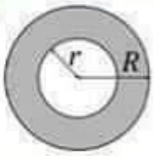
PYTHAGOREAN THEOREM

$a^2 + b^2 = c^2$
$c = \sqrt{a^2 + b^2}$



CIRCULAR RING

$A = \pi(R^2 - r^2)$

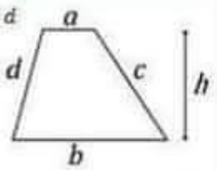


SPHERE

$S = 4\pi r^2$
$V = \frac{4\pi r^3}{3}$

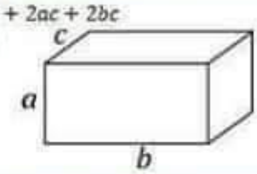
TRAPEZOID

$P = a + b + c + d$
$A = h\frac{a + b}{2}$



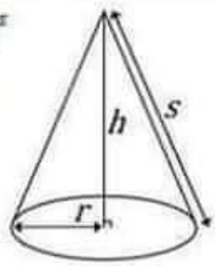
RECTANGULAR BOX

$A = 2ab + 2ac + 2bc$
$V = abc$



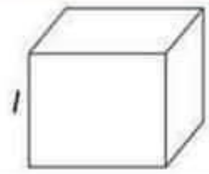
RIGHT CIRCULAR CONE

$A = \pi r^2 + \pi rs$
$s = \sqrt{r^2 + h^2}$
$V = \frac{1}{3}\pi r^2 h$



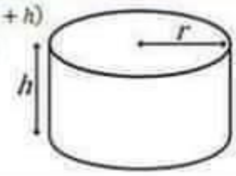
CUBE

$A = 6l^2$
$V = l^3$



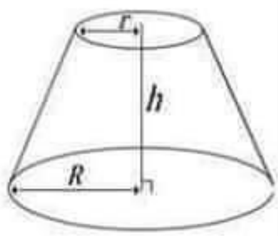
CYLINDER

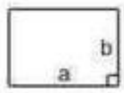
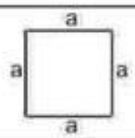
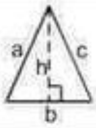
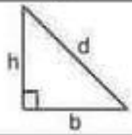
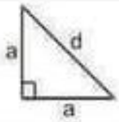
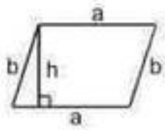
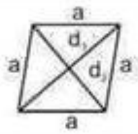
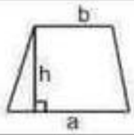
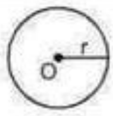
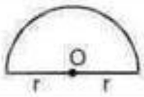
$A = 2\pi r(r + h)$
$V = \pi r^2 h$



FRUSTUM OF A CONE

$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$



Name	Figure	Perimeter	Area
Rectangle		$2(a+b)$	ab
Square		$4a$	a^2
Triangle		$a+b+c=2s$	1 = $\frac{1}{2} \times b \times h$ 2 = $\sqrt{s(s-a)(s-b)(s-c)}$
Right triangle		$b+h+d$	$\frac{1}{2}bh$
Equilateral triangle		$3a$	1. $\frac{1}{2}ah$ 2. $\frac{\sqrt{3}}{4}a^2$
Isosceles right triangle		$2a+d$	$\frac{1}{2}a^2$
Parallelogram		$2(a+b)$	ah
Rhombus		$4a$	$\frac{1}{2}d_1d_2$
Trapezium		Sum of its four sides	$\frac{1}{2}h(a+b)$
Circle		$2\pi r$	πr^2
Semicircle		$\pi r + 2r$	$\frac{1}{2}\pi r^2$
Ring (shaded region)		----	$\pi(R^2 - r^2)$
Sector of a circle		$l + 2r$ where $l = (\theta/360) \times 2\pi r$	$\theta/360° \times \pi r^2$

Mathematics Symbols

+	plus
−	minus
×	multiplied by
÷	divided by
±	plus or minus
>	is greater than
<	is less than
=	is equal to
≠	is not equal to
~	is similar to
≅	is congruent to
∞	infinity
≥	is greater than or equals
≤	is less than or equals
⇔	is equivalent to
⇒	implies
θ	theta
∅	empty set
Δ	triangle or delta
∀	for all
π	pi; 3.14159
∫	integral
∩	intersection of two sets
∪	union of two sets
!	factorial
∴	therefore
√	Square root of
⊥	perpendicular
∃	exists
%	percent
$\overleftrightarrow{AB}$	line AB
$\overline{AB}$	segment AB
$\overrightarrow{AB}$	ray AB
∟	right angle
∠	angle
Σ	sum of
{ }	braces (grouping)
[ ]	brackets
( )	parentheses (grouping)


Jan
Feb
Dec
Mar
Nov
Apr
Oct
May
Sep
Jun
Aug
Jul
31
28/29
31
30
31
30
31
© BRIGHTSIDE

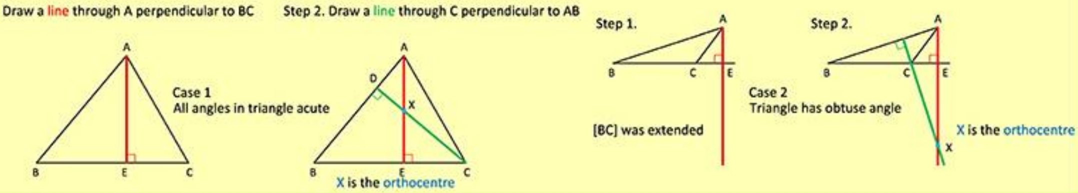
GEOMETRY CONSTRUCTIONS 2

22. Find the orthocentre of the triangle ABC

Note: The orthocentre is the point of intersection of the perpendiculars from the vertices to the opposite sides (a side may have to be extended)

Step 1. Draw a line through A perpendicular to BC

Step 2. Draw a line through C perpendicular to AB



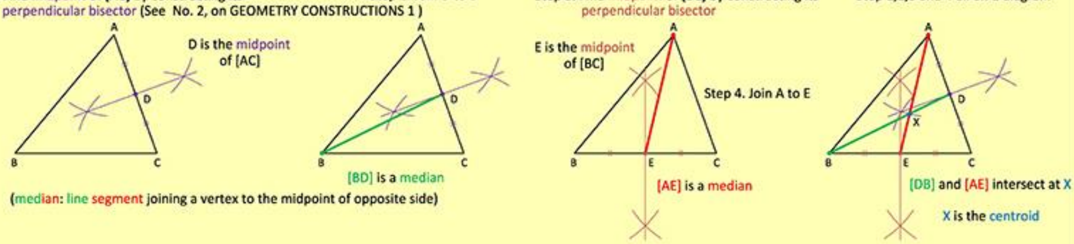
21. Find the centroid of the triangle ABC (centroid: the point of intersection of the medians)

Step 1. Find midpoint of [AC] by constructing its perpendicular bisector (See No. 2, on GEOMETRY CONSTRUCTIONS 1)

Step 2. Join D to B

Step 3. Find midpoint of [BC] by constructing its perpendicular bisector

Step 1,2,3 and 4 all on 1 diagram



(median: line segment joining a vertex to the midpoint of opposite side)

20. Construct the parallelogram ABCD where |AB| = 6 cm, |AD| = 4 cm, | ∠ BAD | = 30°

Step 1. Rough Diagram

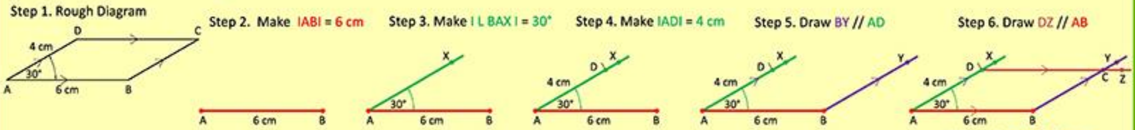
Step 2. Make |AB| = 6 cm

Step 3. Make | ∠ BAX | = 30°

Step 4. Make |AD| = 4 cm

Step 5. Draw BY // AD

Step 6. Draw DZ // AB

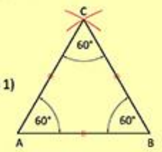


ABCD is the parallelogram

18. Construct an angle of 60°, without using a protractor or setsquare

Note: An equilateral triangle has equal sides and equal angles (of 60°)

Method (See No. 10, on GEOMETRY CONSTRUCTIONS 1)
Construct a triangle ABC where |AB| = |BC| = |AC|



19. Draw a tangent to circle, centre O, at X

Step1. Join O to X

Step2. Draw a line perpendicular to OX through X

Note: A tangent is perpendicular to the radius at the point of contact

17. Find the incentre and draw the incircle of the triangle ABC (incentre: the point of intersection of the bisectors of the angles)

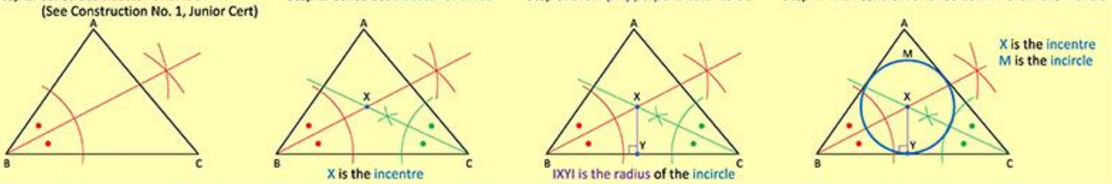
(incircle: circle inside a triangle, just touching all 3 sides)

Step 1. Construct bisector of ∠ ABC (See Construction No. 1, Junior Cert)

Step 2. Construct bisector of ∠ ACB

Step 3. Draw [XY] perpendicular to BC

Step 4. With centre X and radius |XY| draw the incircle



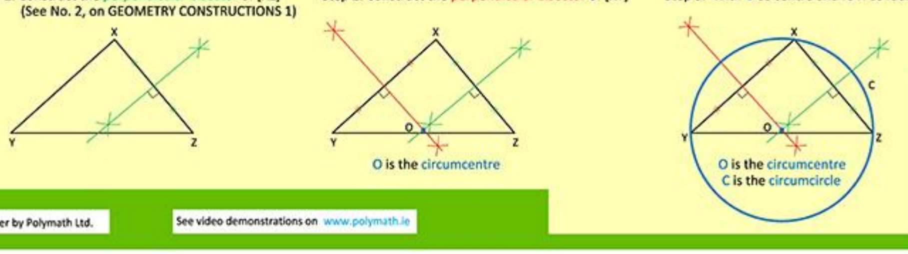
16. Find the circumcentre and draw the circumcircle of the triangle XYZ. (circumcentre: point of intersection of the perpendicular bisectors of the sides)

(circumcircle: a circle which goes through the 3 vertices of a triangle)

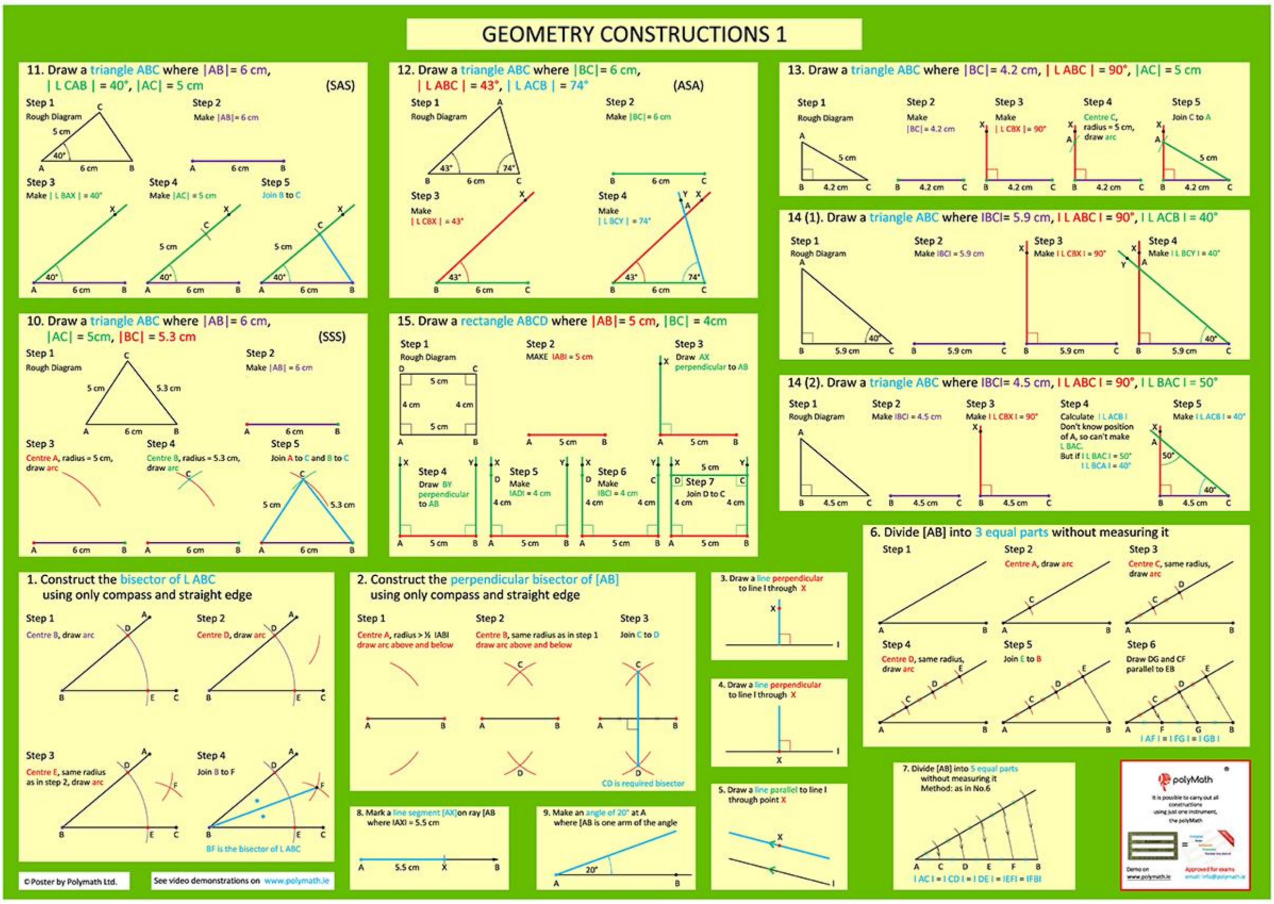
Step 1. Construct the perpendicular bisector of [XZ] (See No. 2, on GEOMETRY CONSTRUCTIONS 1)

Step 2. Construct the perpendicular bisector of [XY]

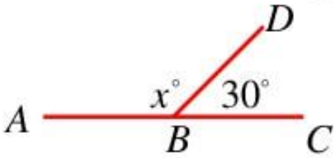
Step 3. With O as centre and |OY| as radius, draw the circumcircle



See video demonstrations on www.polymath.ie



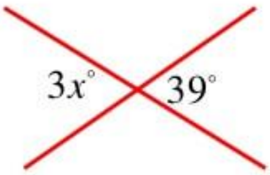
Angle Theorems



Angles in a straight line add up to 180°

$x + 30 = 180$ (straight $\angle ABC = 180^\circ$)

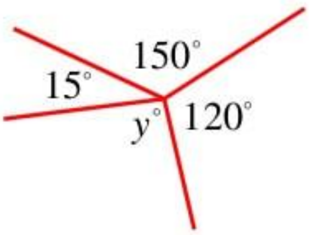
$x = 150$



Vertically opposite angles are equal

$3x = 39$ (vertically opposite $\angle$'s are $=$)

$x = 13$

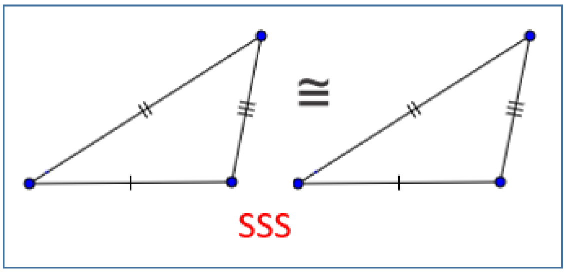


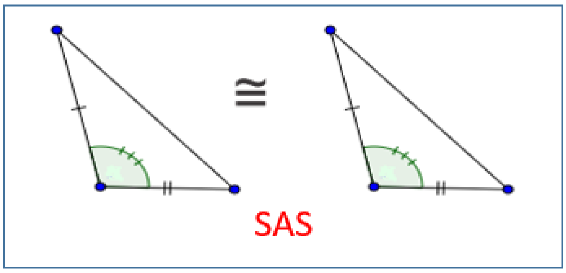
Angles about a point equal 360°

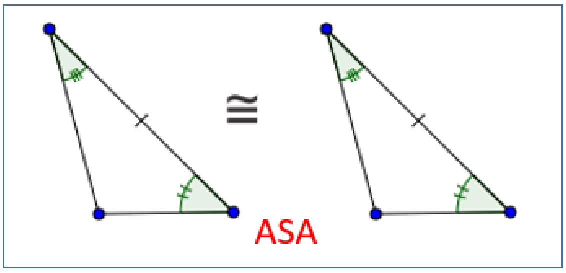
$y + 15 + 150 + 120 = 360$ (revolution $= 360^\circ$)

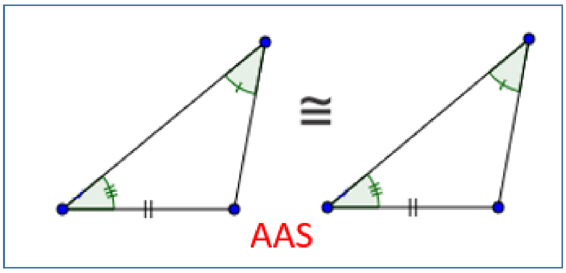
$y = 75$

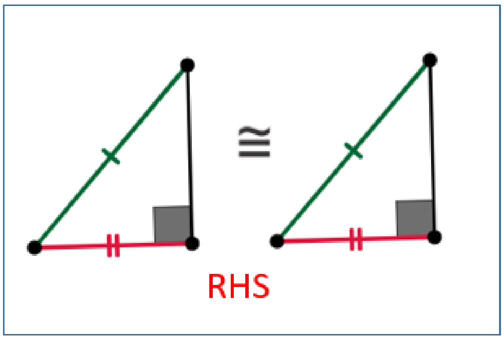
Rules for Triangle Congruency

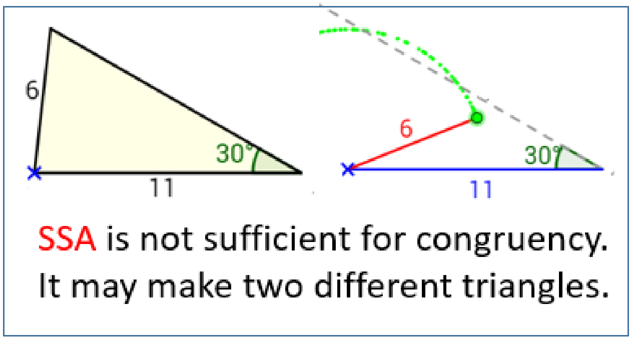


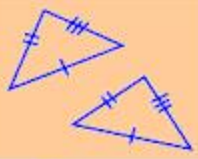
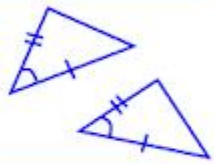
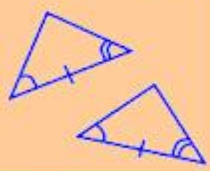
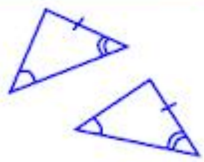










Methods for Proving (Showing) Triangles to be Congruent		
SSS **Side-Side-Side**	If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent. (For this method, the sum of the lengths of any two sides must be greater than the length of the third side, to guarantee a triangle exists.)	
SAS **Side-Angle-Side**	If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. (The included angle is the angle formed by the sides being used.)	
ASA **Angle-Side-Angle**	If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. (The included side is the side between the angles being used. It is the side where the rays of the angles would overlap.)	
AAS **Angle-Angle-Side**	If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. (The non-included side can be either of the two sides that are not between the two angles being used.)	
HL **Hypotenuse-Leg**	If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent. (Either leg of the right triangle may be used as long as the corresponding legs are used.)